Let's look at a specific example. Consider two Einstein solids, A and B, that are "weakly coupled" so that they can exchange energy (but with the total energy fixed). Suppose (as in Figure 2.5) that the numbers of oscillators in the two solids are $N_A = 300$ and $N_B = 200$, and that they are sharing 100 units of energy: $q_{\text{total}} = 100$. Table 3.1 lists the various macrostates and their multiplicities. Now, however, I have also included columns for the entropy of solid A, the entropy of solid B, and the total entropy (which can be obtained either by adding S_A and S_B, or by taking the logarithm of Ω_{total}).

Figure 3.1 shows a graph of S_A, S_B, and S_{total} (in units of Boltzmann's constant), for the same parameters as in the table. The equilibrium point is at $q_A = 60$, where S_{total} reaches its maximum value. At this point, the tangent to the graph of S_{total} is horizontal; that is,

$$\frac{\partial S_{\text{total}}}{\partial q_A} = 0 \qquad \text{or} \qquad \frac{\partial S_{\text{total}}}{\partial U_A} = 0 \qquad \text{at equilibrium.} \tag{3.1}$$

(Technically it's a *partial* derivative because the number of oscillators in each solid is being held fixed. The energy U_A is just q_A times a constant, the size of each unit of energy.) But the slope of the S_{total} graph is the sum of the slopes of the S_A and S_B graphs. Therefore,

$$\frac{\partial S_A}{\partial U_A} + \frac{\partial S_B}{\partial U_A} = 0 \qquad \text{at equilibrium.} \tag{3.2}$$

The second term in this equation is rather awkward, with B in the numerator and A in the denominator. But dU_A is the same thing as $-dU_B$, since adding a bit of energy to solid A is the same as subtracting the same amount from solid B.

q_A	Ω_A	S_A/k	q_B	Ω_B	S_B/k	Ω_{total}	S_{total}/k
0	1	0	100	2.8×10^{81}	187.5	2.8×10^{81}	187.5
1	300	5.7	99	9.3×10^{80}	186.4	2.8×10^{83}	192.1
2	45150	10.7	98	3.1×10^{80}	185.3	1.4×10^{85}	196.0
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
11	5.3×10^{19}	45.4	89	1.1×10^{76}	175.1	5.9×10^{95}	220.5
12	1.4×10^{21}	48.7	88	3.4×10^{75}	173.9	4.7×10^{96}	222.6
13	3.3×10^{22}	51.9	87	1.0×10^{75}	172.7	3.5×10^{97}	224.6
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
59	2.2×10^{68}	157.4	41	3.1×10^{46}	107.0	6.7×10^{114}	264.4
60	1.3×10^{69}	159.1	40	5.3×10^{45}	105.3	6.9×10^{114}	264.4
61	7.7×10^{69}	160.9	39	8.8×10^{44}	103.5	6.8×10^{114}	264.4
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
100	1.7×10^{96}	221.6	0	1	0	1.7×10^{96}	221.6

Table 3.1. Macrostates, multiplicities, and entropies of a system of two Einstein solids, one with 300 oscillators and the other with 200, sharing a total of 100 units of energy.