Physics 3180 (Thermal Physics) Name ____________________

spring 2020

Problem Set 5

(due Friday, February 21)

1. **For formal written solution:** Write a self-contained narrative describing the energy and entropy flows, and the efficiency, of a large steam-turbine power plant that absorbs heat at a rate of 3 GW from a heat source whose temperature is 700 K, while expelling waste heat to the environment at 300 K. Some specific questions for you to ask and answer are listed in parts (a) through (d) below, but your narrative should flow smoothly, motivating each of these questions and discussing the implications along the way. Be sure to start with a brief introduction that states your main goal(s).

   (a) During one second of operation, how much entropy is lost by the heat source (due to heat absorption by the water/steam)?

   (b) Assuming that the environment must gain at least this much entropy, how much waste heat must be expelled?

   (c) Use your answer to (b) to determine the maximum possible efficiency of this power plant. Check that you get the same answer from the Carnot formula, $1 - (T_c/T_h)$.

   (d) The actual efficiency of this power plant is 35%. How much *new* entropy does it therefore create each second? How would you summarize the relationship between entropy and efficiency for this power plant?

Additional problems:

2. As long as the temperature isn't too low, the heat capacities of most solids can be modeled by the formula

$$C_P = a + bT - \frac{c}{T^2},$$

   where a, b, and c are constants. For one mole of aluminum, the formula is fairly accurate if we take $a = 3R$ (the equipartition prediction for C_V), $b = 0.0032$ J/K^2, and $c = 2.28 \times 10^5$ J·K. Use this information to find the change in the entropy of one mole of aluminum as it is heated from 298 K to 600 K. Then add on the tabulated value at 298 K to obtain the entropy at 600 K.

3. Imagine a liter of air in a cylinder with a piston, initially at room temperature and atmospheric pressure. You then pull the piston outward *instantaneously*, increasing the volume available to the air by 1%. The air then expands freely to fill the additional space, eventually reaching a new equilibrium.

   (a) What are the work done on the air, heat added to the air, and total change in the air's energy during this process?

   (b) What is the value of $-P\,dV$ for this process? Why is this *not* equal to the work done on the air?

   (c) Use the thermodynamic identity to determine the change in the entropy of the air during this process, and discuss your result. Can you calculate dS using a different method and compare?

4. Consider an Einstein solid with two oscillators and five units of energy. Using the same type of reasoning as on pages 117–118 in your text, determine the chemical potential of this system, in terms of ϵ. Then repeat for an Einstein solid with four oscillators and four units of energy.

5. Problem 3.37, pages 119–120.

6. Problem 3.39, page 121.

7. A heat engine using air as its working substance undergoes the following cyclic process: (i) starting at volume V_0 and pressure P_0, the air is heated at constant volume until the pressure reaches $2P_0$; (ii) the air is then heated at constant pressure, expanding to volume $2V_0$; (ii) finally, the gas is cooled and compressed in such a way that the pressure decreases linearly as a function of volume, returning the gas to its initial state.

   (a) Draw this cycle on a PV diagram.

   (b) Argue that the net work done during the cycle is the area enclosed on your diagram, and find a formula for this area (in terms of V_0 and P_0).

   (c) Use the principles that you learned in Chapter 1 to find a formula for the heat added to the gas during step (i). Repeat for step (ii). (You may assume $f = 5$ for air.)

   (d) What is the efficiency of this engine?

   (e) Where during the cycle is the temperature of the air highest? Lowest? How do these two temperatures compare?

   (f) What is the maximum possible efficiency of an engine operating over this temperature range?

8. Draw an energy-flow diagram, similar to figures 4.1 and 4.4, for an electric space heater. (Hint: It's *simpler* than an engine or a refrigerator!) How would you define the efficiency of such a device? What limits on the efficiency are implied by the laws of thermodynamics? How would you respond to a salesperson who claims that a particular model of electric space heater is twice as efficient as the one you already have? Explain your answers clearly.

9. Modern central air conditioning units typically have a COP of about 3.4 when operating between an outdoor temperature of 95°F and an indoor temperature of 80°F.

   (a) How does this COP compare to the maximum possible COP for this temperature range?

   (b) Suppose that such an air conditioner removes heat from indoors at a rate of 24,000 Btu/hr. If it operates for an effective average of 5 hours/day under these conditions, and you pay 10 cents per kilowatt-hour for electricity, what is your monthly electricity bill (just to run the air conditioner)?

10. Problem 4.15, page 130.