Physics 3180 (Thermal Physics) Name ______________________
spring 2020

Problem Set 3

(due Friday, January 31, 10:30 am)

1. **For formal written solution:** This problem applies Stirling's approximation to the probability for flipping many coins and getting some given number of heads. Although it is divided into parts for your convenience, please don't refer to the parts by letter in your solution. Instead, write a coherent narrative that explains what you're doing (and why) at each stage.

   (a) Use Stirling's approximation to find a simple (but approximate) formula for the probability of getting exactly half heads and half tails when you flip N fair coins (assuming that N is reasonably large). Be sure to simplify your formula as much as possible; the answer you're looking for involves a square root, but has no factorials or combinations or even exponentiation. Evaluate your formula numerically (to at least five significant figures) for $N = 100$, 1000, and 10,000.

   (b) Use Mathematica to find the "exact" answer (in decimal approximation) for $N = 100$, 1000, and 10,000. Compare to your answers to part (a); you should find that they are very close, but not exactly the same. (The Mathematica function for combinations is called `Binomial`. Be sure to quote your exact instruction to Mathematica in your writeup.)

   (c) Sketch a qualitatively accurate graph of the probability of getting N_H heads, as a function of N_H, for fixed N. How high (in terms of N) is the highest point on the graph? What is the total area under the graph? (Explain how you know.)

   (d) Use your answers to parts (a) and (c) to make a rough estimate of the width of the peak in your graph, as a function of N. Discuss how the appearance of the graph changes as N becomes larger and larger, and what this result means in terms of coin flips.

Additional problems:

2. Problem 2.9, page 60. For this exercise you need to use a computer spreadsheet program. In Excel, Numbers, OpenOffice, or Google Docs, the function that computes "combinations" is called `COMBIN`; for example, to compute the number of ways of choosing 20 items from 100, you would enter "`=COMBIN(100,20)`" (use a semicolon instead of a comma in OpenOffice). Do *not* try to compute the combinations explicitly in terms of factorials, since this will generate overflow errors in the next problem. For further hints, see the "computer problems" link at physics.weber.edu/thermal. Please turn in a printout showing the *modified* table and graph—not the original version that's already printed in the textbook.

3. Use a computer to produce a table and graph, similar to those in Figure 2.5, for the case where one Einstein solid contains 150 oscillators, the other contains 250 oscillators, and there are 100 units of energy in total. Turn in a printout of the table and the graph, highlighting the most probable and least probable macrostates. What is the probability of the most probable macrostate? Of the least probable macrostate?

4. Problem 2.12, page 61.

5. Problem 2.13, page 62.

6. Problem 2.18, page 64.

7. Problem 2.26, page 72.

8. Consider again the system of N coins from Problem 1. Treating each coin as a two-state system (i.e., neglecting all properties other than its two possible orientations), what is the entropy of this system, as a function of N, if the orientations of the coins are all unspecified? Evaluate the entropy numerically for $N = 100$, 1000, and 10,000. Please express each answer both as a unitless number (neglecting the factor of Boltzmann's constant) and in the traditional units of J/K.

9. Consider again the system from Problem 3 above: two Einstein solids, with $N_A = 250$, $N_B = 150$, and $q_{\text{total}} = 100$. Compute the entropy of the most likely macrostate and of the least likely macrostate. Also compute the entropy over long time scales, assuming that *all* microstates are accessible. Discuss your results briefly. (Please neglect the factor of Boltzmann's constant in these entropy calculations; for systems this small it is best to think of entropy as a unitless number.)

10. Problem 2.32, page 79.