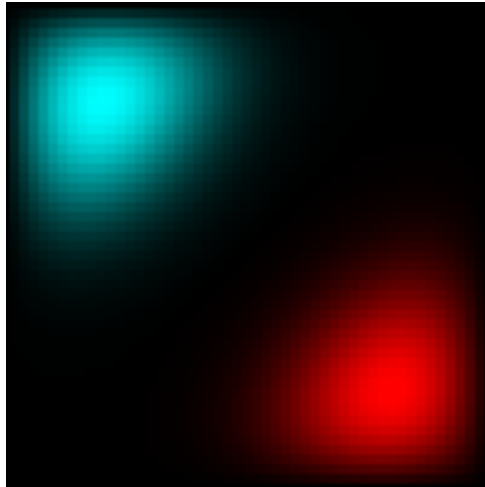


Easy Examples of Emerging Entanglement

Dan Schroeder, Weber State University, 26 July 2017

1. Bound states



2. Scattering

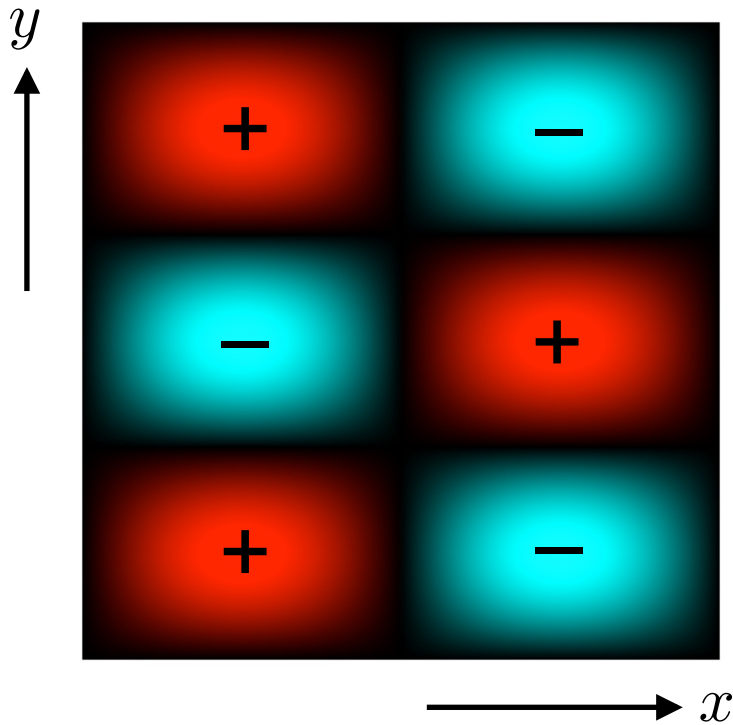


Bonus: Animated web apps at physics.weber.edu/schroeder/software

Context: Any course that covers quantum wavefunctions in more than one dimension

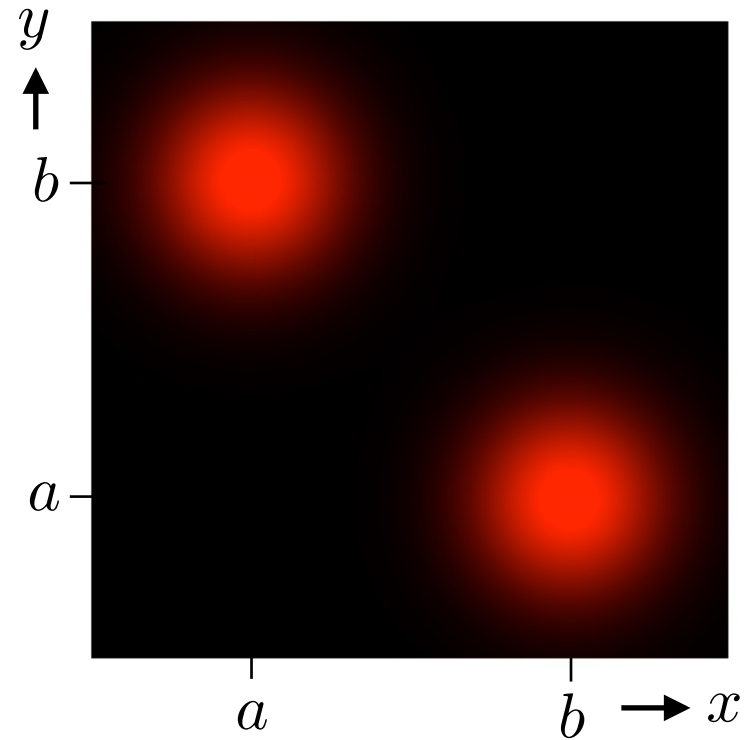
Two-dimensional wavefunctions

Separable



$$\psi(x, y) = \sin(2\pi x) \sin(3\pi y)$$

Nonseparable

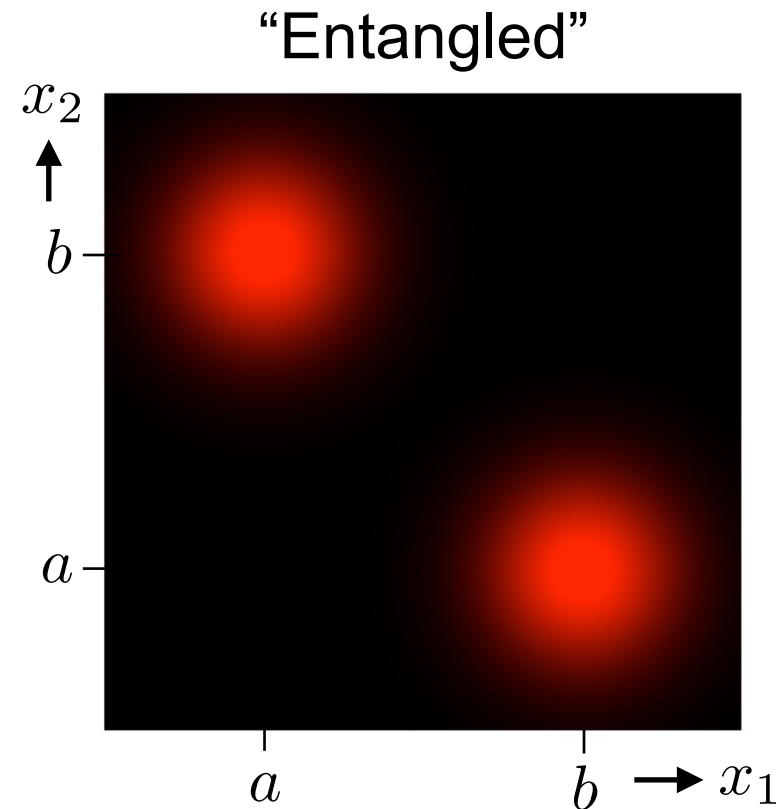


$$\psi(x, y) \neq f(x) g(y)$$

Two particles in one dimension

- Impossible to describe the state of one particle without referring to the other.
- Measuring the position of one particle changes the probability distribution for the other.

But when do entangled states actually occur?

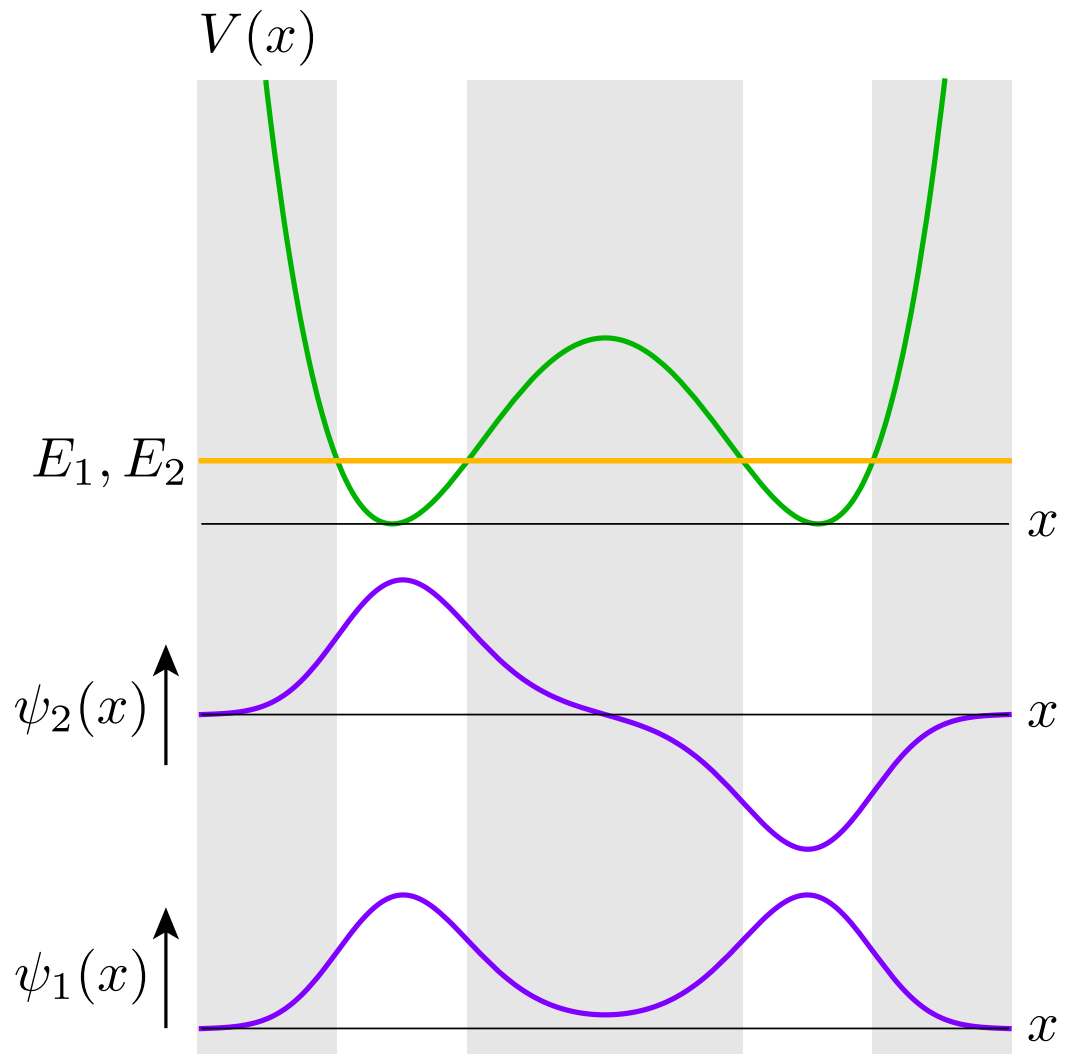


$$\psi(x_1, x_2) \neq f(x_1) g(x_2)$$

Example 1: Energy eigenstates of a bound system

1D Double Well

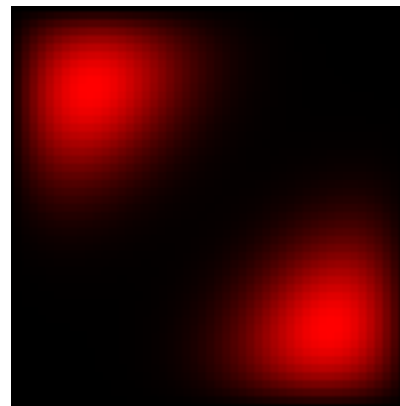
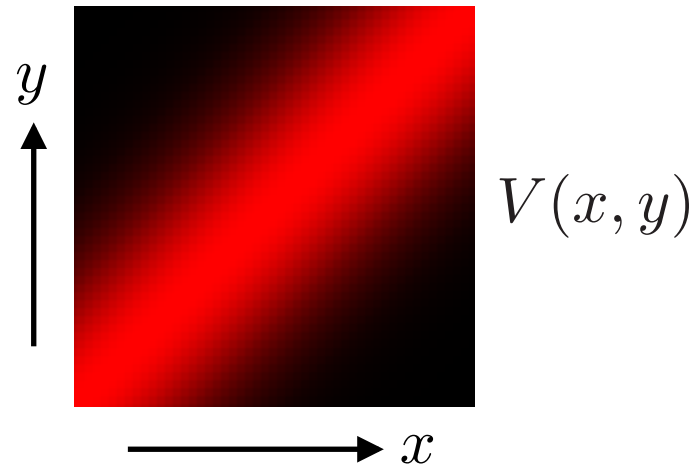
- Easy to sketch wavefunctions qualitatively
- Low-energy states have two separated peaks
- Examples: H_2 , NH_3 , fabricated nano-structures (qubits!)



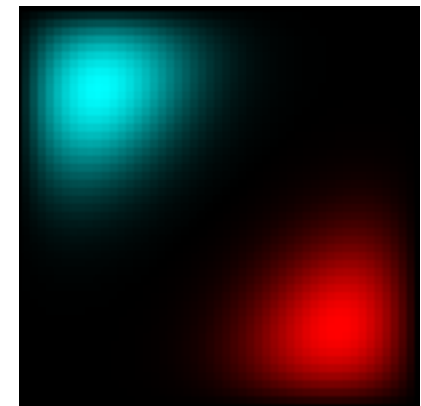
Example 1: Energy eigenstates of a bound system

2D Double Well

- Square infinite square well
- Add barrier along diagonal
- Again, two separated peaks



$\psi_1(x, y)$

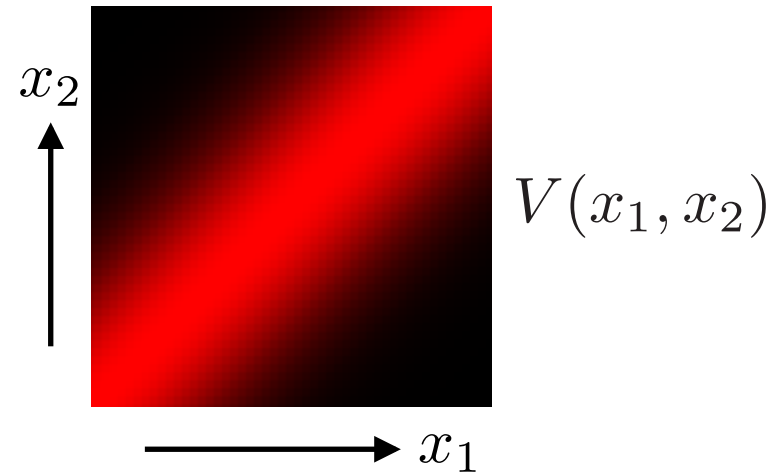
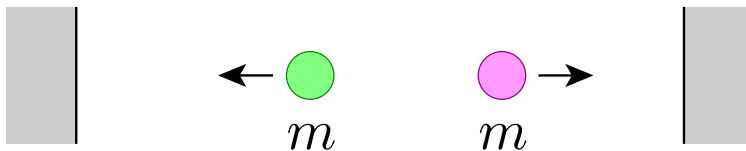


$\psi_2(x, y)$

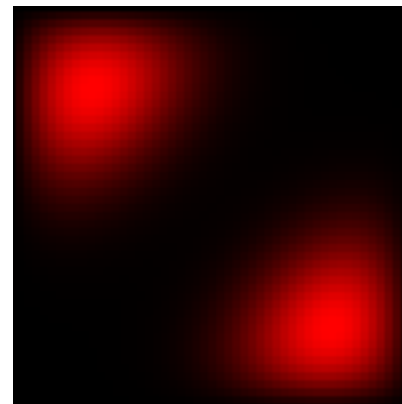
Example 1: Energy eigenstates of a bound system

Two particles in 1D well

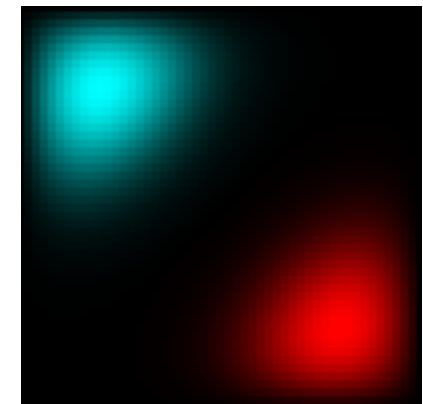
Repulsive interaction



- Equal masses but distinguishable
- Same math, so same solutions
- **Entangled states!**



$\psi_1(x_1, x_2)$

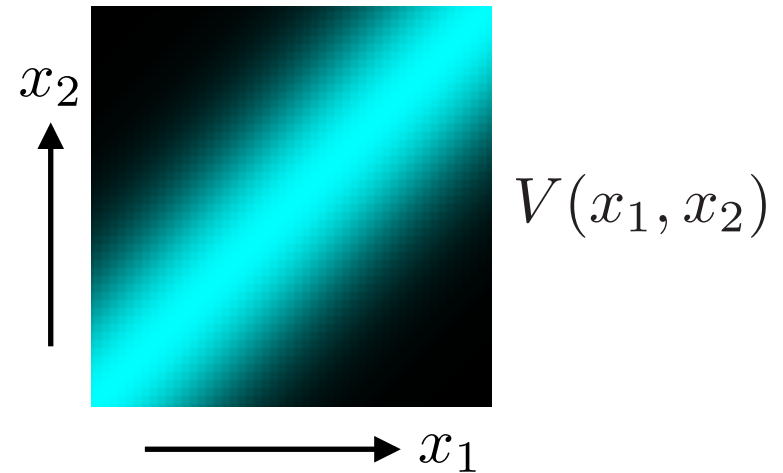
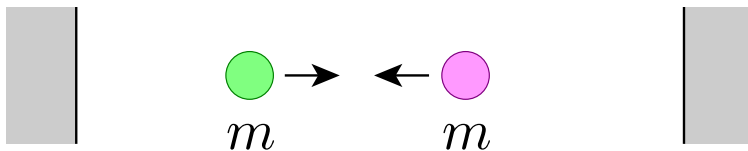


$\psi_2(x_1, x_2)$

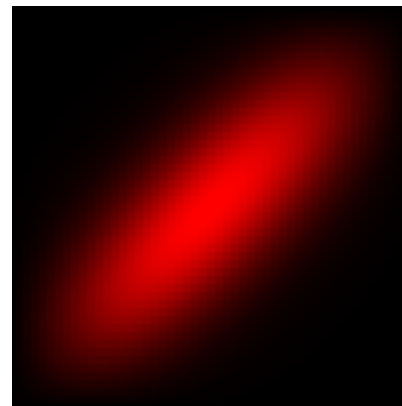
Example 1: Energy eigenstates of a bound system

Two particles in 1D well

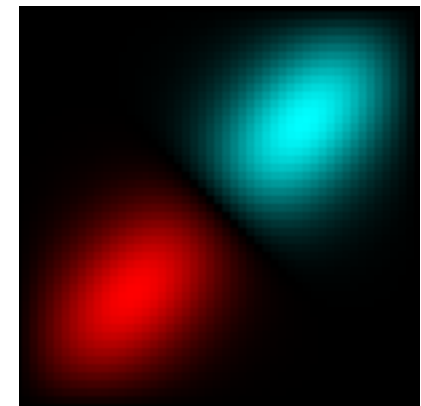
Attractive interaction



- Particles tend to form a bound state
- **Still entangled!**



$\psi_1(x_1, x_2)$



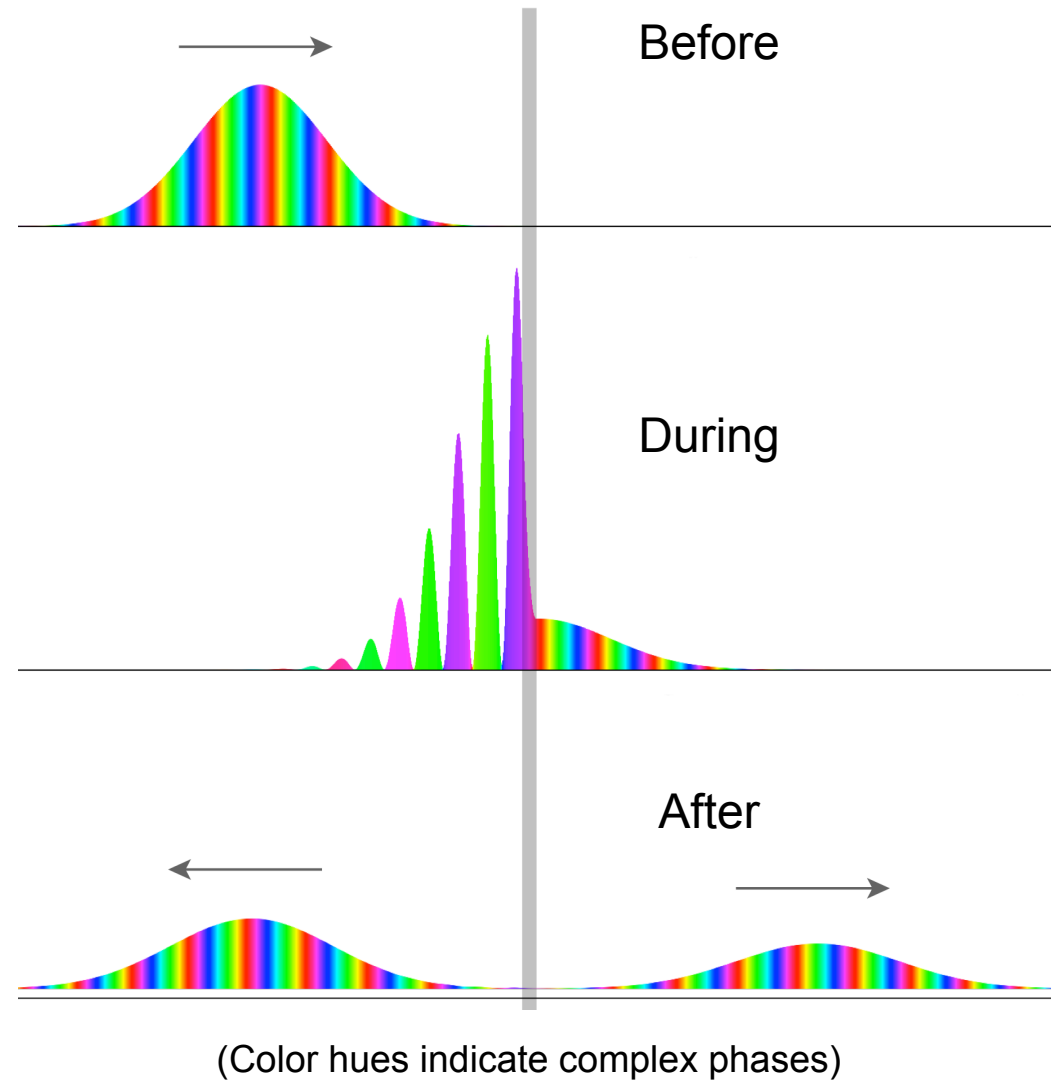
$\psi_2(x_1, x_2)$

Example 2: Wavepacket scattering

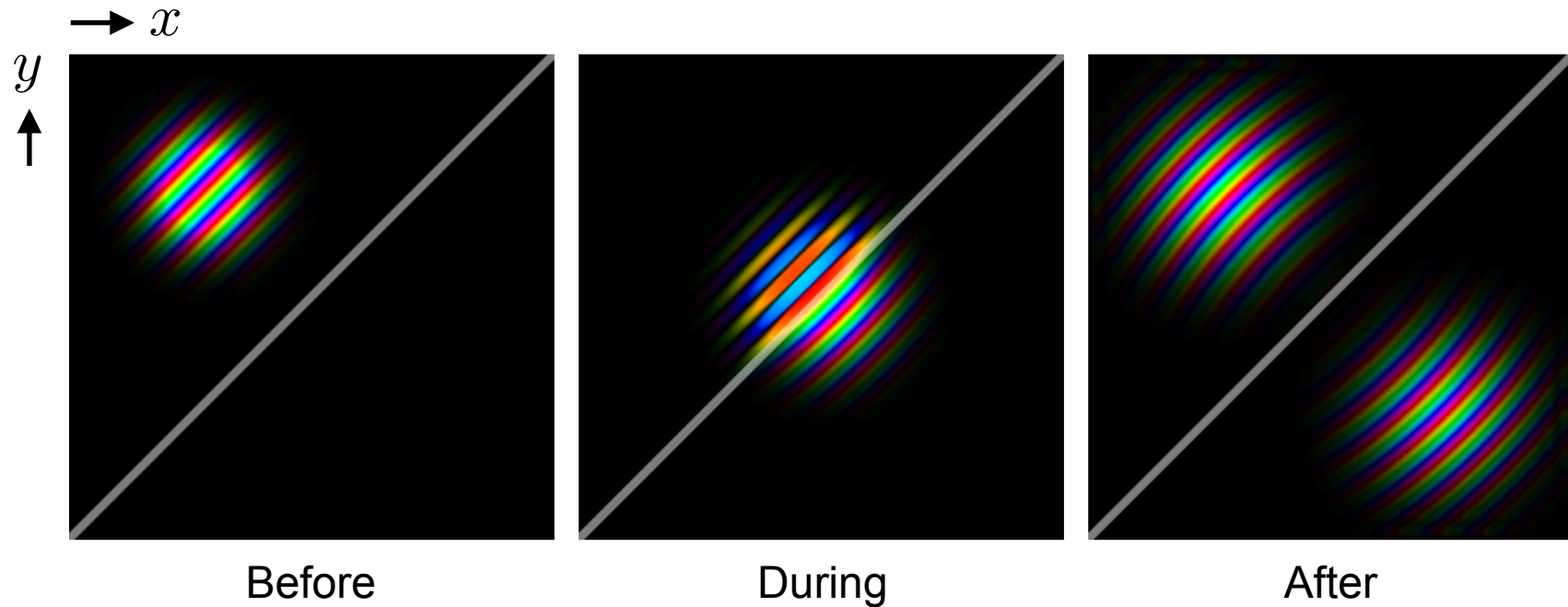
In One Dimension

Wavepacket splits into 2 pieces due to:

- Boundary reflections
- Tunneling
- Filtering



Example 2: Wavepacket scattering In Two Dimensions

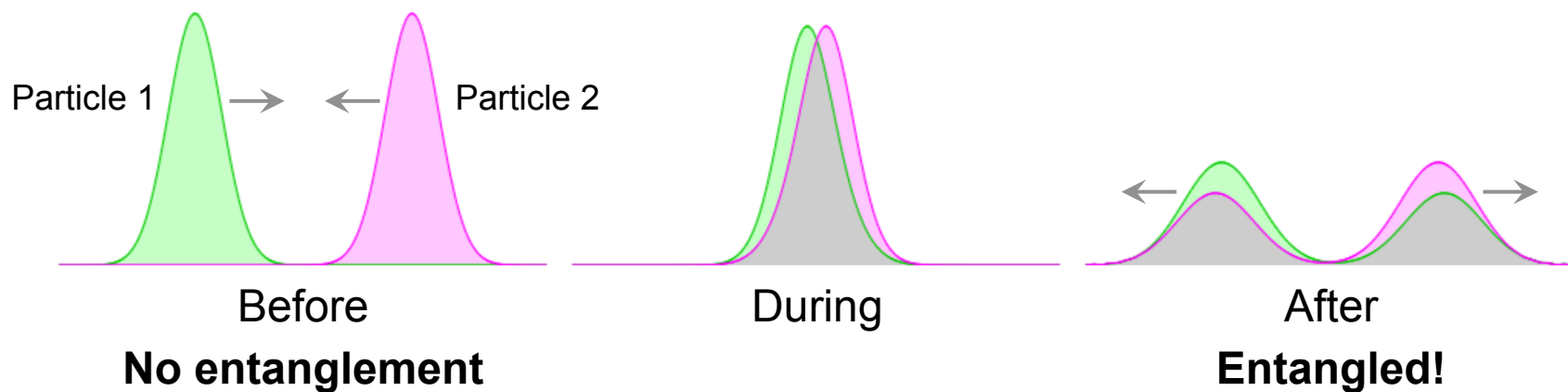
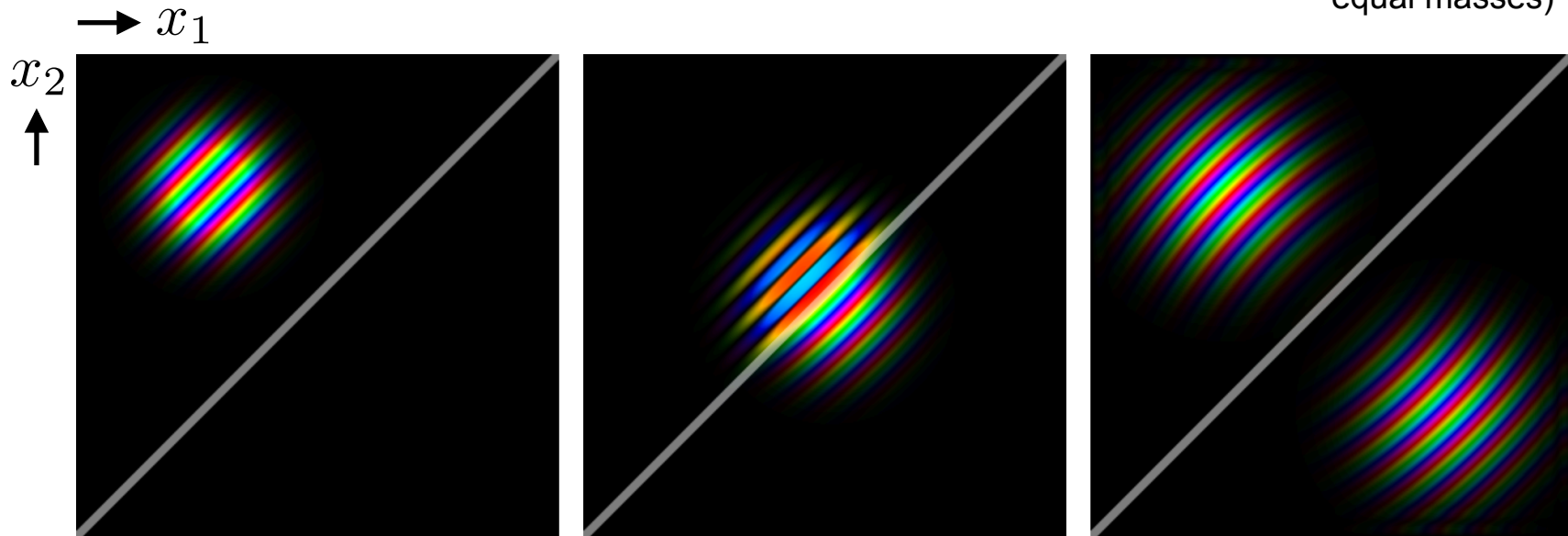


Conceptually identical to the 1D case!

Example 2: Wavepacket scattering

Two particles in one dimension

(Distinguishable,
equal masses)



DISCUSSION OF PROBABILITY RELATIONS BETWEEN SEPARATED SYSTEMS

By E. SCHRÖDINGER

[Communicated by Mr M. BORN]

[*Received* 14 August, *read* 28 October 1935]

1. When two systems, of which we know the states by their respective representatives, enter into temporary physical interaction due to known forces between them, and when after a time of mutual influence the systems separate again, then they can no longer be described in the same way as before, viz. by endowing each of them with a representative of its own. I would not call that *one* but rather *the* characteristic trait of quantum mechanics, the one that enforces its entire departure from classical lines of thought. By the interaction the two representatives (or ψ -functions) have become entangled. To disentangle them we must gather further information by experiment, although we knew as much as anybody could possibly know about all that happened. Of either system, taken separately, all previous knowledge may be entirely lost, leaving us but one

[Math. Proc. Camb. Phil. Soc. **31**(4), 555-563]

Conclusions

- Interactions *generically* create entanglement.
- We can teach this! Examples are a small conceptual step from familiar 1D scenarios.
- In an upper-division course we can also teach algorithms for a quantitative numerical treatment.

Paper: “Entanglement isn’t just for spin,” arXiv:1703.10620
[physics.ed-ph] (submitted to AJP)

Web apps: <http://physics.weber.edu/schroeder/software>

Notes for Easy Examples of Emerging Entanglement

Dan Schroeder, Weber State University

1. I'd like to tell you how you can naturally introduce the concept of entanglement into any course that covers wavefunctions in more than one dimension: Not just an upper-division quantum mechanics course, but also a sophomore-level "modern physics" course and even some introductory physics courses. I'll show two examples: one involving bound states, and one involving wavepacket scattering.
2. As background, let's consider some wavefunctions for a single particle in two-dimensional space. On the left I've drawn one of the energy eigenstates of a square infinite square well, using red for positive values and cyan for negative values. What I want to emphasize about this wavefunction is that it's separable into a function of x times a function of y . But the vast majority of two-dimensional wavefunctions are not separable. I've drawn just one example at right, a double-peaked wavefunction in which the particle has a 50-50 chance of being in either of two locations. There's no way to write this wavefunction as a function of x times a function of y .
3. But any wavefunction for a single particle in two dimensions is also an allowed wavefunction for a system of *two* particles in *one* spatial dimension. So here I've relabeled x and y as x_1 and x_2 , so we now have a plot in two-particle configuration space. And this particular wavefunction is what we would call an entangled state: It can't be written as a product of a function of x_1 times a function of x_2 ; or equivalently, there's no way to fully describe the state of one particle without also referring to the other; or equivalently, if you measure some observable quantity for one particle, that changes the probability distribution for a subsequent measurement on the other particle. So, for instance, if you measure the position of particle 1 and find it near b , you've also now given particle 2 no choice but to be near a . So we can easily draw pictures of entangled two-particle states. But when do these states actually occur, and what causes them to occur, dynamically? I'll give two examples.
4. To get started with my first example, let's consider a single particle in a one-dimensional double-well potential. If its ground-state energy is at the level shown here, then it's easy to sketch the ground-state wavefunction, which must have a peak in each of the two classically allowed regions, and a dip in between, but no nodes. The first excited state has a very slightly higher energy (indistinguishable in the picture), with a node in the middle. Incidentally, if you're not already covering this system you should, because there are important real-world examples and it's a great first example of a qubit. But that's another lecture.
5. Now let's consider a double well for a two-dimensional system, still just one particle. This is a square infinite square well, but I've added a smooth potential barrier centered on the main diagonal, dividing the square into a double well. Therefore the ground state wavefunction has a bump on each side of the barrier, and the first excited state has a slightly higher energy, with one bump being negative, and a node along the diagonal. Easy, right?

6. But those same functions apply to two particles in one spatial dimension. So now we have two distinguishable, equal-mass particles in a one-dimensional box, with a repulsive force between them. They want to get far away from each other, on opposite sides of the box, but they have no preference for which is on which side. So the bright spot at the upper-left puts particle 1 on the left and particle 2 on the right, and the bright spot at the lower-right interchanges them. This is an entangled state, very similar to the one that I drew a few slides ago.
7. It's also fun to switch the sign of the interaction so the particles attract each other. Now they tend to form a bound state with each other. The ground state spreads them both over the width of the box but always near each other; the first excited state puts them jointly on one side or the other of the box, with a node in the middle.
8. My second example involves wavepacket scattering, so let's again review the one-dimensional case. If I aim a wavepacket at a fixed potential barrier, we get a pretty interference pattern during the interaction and then, for suitably chosen barrier strengths and widths, the wavepacket splits into a reflected piece and a transmitted piece.
9. I can also do this in two dimensions. The left image shows my incoming wavepacket, aimed at a narrow barrier running along the diagonal. Again we get a pretty interference pattern during the interaction, and the wavepacket splits into a reflected piece and a transmitted piece. It's conceptually identical to the one-dimensional case.
10. But again, I can reinterpret the same pictures for a system of two particles in one dimension. Now particle 1 starts at the left and moves to the right, while particle 2 starts at the right and moves to the left. I've drawn their probability distributions below. They interact with a short-range repulsive force, and either pass through each other or reflect, so that afterwards each particle can be in two different regions, but if one of them is, say, transmitted, then so is the other. So they've become entangled.
11. Of course Schrödinger knew all this, and said it quite well back in 1935, in the paper in which he coined the term "entangled" (in English!). But we don't typically teach these ideas to beginning students, and in my opinion we should.
12. The conclusions speak for themselves! Also, most of what's in this talk is in a paper that I've submitted to AJP and posted on arXiv. Finally, you (and your students) may enjoy playing with the web apps that I used to draw the pictures.