Definitions and Conventions for Relativity

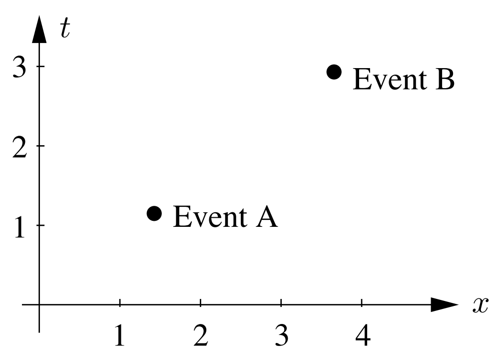
1. Reference frame: a gridwork or “jungle-gym” of meter sticks and synchronized clocks, or any equivalent system of measuring positions and times of events.

2. Clock synchronization: To verify that clocks are synchronized, observers agree to send signals to each other when their clocks reach a certain time. If the signals are also *received* at the same time by both, the clocks are synchronized. (Note: the signals must travel the same speed in both directions; since light always travels with the same speed, it’s safe to use light for the signals.)

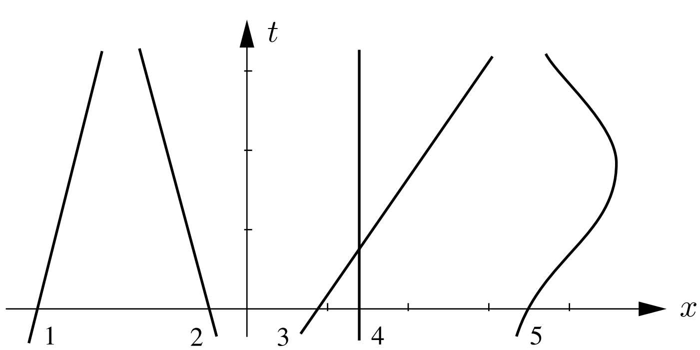
3. Inertial: An *inertial* reference frame is one in which Newton’s first law is valid: isolated objects move in straight lines at constant speeds. If two reference frames are moving at constant velocity with respect to each other, and one is inertial, then so is the other.

4. Event: anything that happens at a localized place and time, such as the explosion of a firecracker, the collision of two billiard balls, or the emission or absorption of a light pulse.

5. Spacetime diagram: a graph of x and t depicting events as measured in some inertial reference frame. By convention, the t axis is taken to be vertical. The axes are calibrated such that one second of time and one light-second of distance are represented by equal distances on the diagram. Note that, if the diagram is drawn for some *other* inertial reference frame (moving with respect to the first), it may look quite different.



6. Worldline: a path on a spacetime diagram, representing all events that happen to a single particle. The faster the particle is moving, the *shallower* the slope of its worldline. The worldline of a light-pulse is inclined at 45°.



Worldline 1: a particle moving slowly to the right
Worldline 2: a particle moving slowly to the left
Worldline 3: a particle moving quickly to the right
Worldline 4: a particle at rest
Worldline 5: a particle moving to the right, turning around, and going back to the left

7. Three kinds of time: The time between two events can be measured in three different (though sometimes overlapping) ways:

Coordinate time: the time between two events as measured by a pair of synchronized clocks, one at each event, attached to some inertial reference frame. (If the events happen at the same place, a single clock is sufficient. You can avoid needing multiple clocks by relying on light signals from the events, but you need to carefully account for the light travel time.) Symbol: Δt. Generally, when reference frames are moving with respect to each other, they will measure different values of Δt for the same pair of events.

Proper time: the time between two events as measured by a single clock that is present at both events. Symbol: $\Delta\tau$. Generally, different clocks that travel different worldlines between the two events will measure different values of $\Delta\tau$.

Spacetime interval: same as proper time, but the clock must be inertial: a reference frame attached to it must be an inertial frame (so the clock must be moving at constant velocity with respect to any other inertial frame). Symbol: Δs. For a given pair of events there is only one Δs.

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The Metric Equation: The coordinate time between two events, the distance Δx between them (in the same reference frame), and the spacetime interval are related by the equation

$$(\Delta s)^2 = (\Delta t)^2 - (\Delta x)^2.$$

This equation assumes that Δx is measured in units of the distance that light travels during one unit of time; for instance, if Δt and Δs are in seconds, then Δx must be in light-seconds. If Δx is measured in other units such as meters, the equation reads

$$(\Delta s)^2 = (\Delta t)^2 - (\Delta x/c)^2.$$

(Since c equals one light-second per second, this equation reduces to the previous one if Δx is measured in light-seconds.) If one of the two events is at the origin, so that $\Delta t = t_A$ and $\Delta x = x_A$, then the set of all events with a given Δs defines a hyperbola, which intersects the t axis at Δs, as shown below.