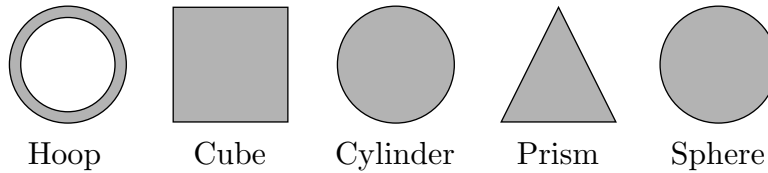


**Problem Set 8**  
(due Monday, March 8)

1. A banked circular highway curve is designed for traffic moving at 30 mph. (So at this speed, no friction is necessary to keep a car on the road.) The radius of the curve is 150 m. But you're in a hurry, so you try to take the curve at 55 mph. Unfortunately, the road surface is wet, so the coefficient of friction between your tires and the road is only 0.20. Will you crash? (Hint: To answer this question you need to solve *two* constrained-motion problems—one to find the banking angle and a second to determine what happens to you. In each case, start by drawing a force diagram as viewed from the front or rear of the car. Choose your axes so that  $\vec{a}$  points along one of them.)
2. Estimate the gravitational force between you and a person sitting 2 m away from you. (You should find that the attraction is negligible. As Einstein once said, "Gravity cannot be held responsible for people falling in love.")
3. The planet Mars has a satellite (moon), called Phobos, which travels in an approximately circular orbit of radius  $9.4 \times 10^6$  m with a period of 7 hours, 39 minutes. From this information and Newton's law of gravity, determine the mass of Mars.
4. Communication satellites are placed in circular orbits above the earth's equator at such a height that they orbit exactly once every 24 hours, and therefore appear to remain fixed in our sky. How far above earth's surface is such a satellite? Please solve this problem from first principles (not from Kepler's third law).
5. A phonograph turntable rotating at  $33\frac{1}{3}$  rpm slows down and stops in 30 s after the motor is turned off. (a) What is its average angular acceleration in  $\text{rev}/\text{min}^2$ ? What is it in  $\text{rad}/\text{s}^2$ ? (b) How many revolutions did it make in this time?
6. A wheel of radius 0.25 m rolls without slipping on a horizontal surface. Its initial speed is 42 m/s and it slows down uniformly, coming to a stop after rolling 225 m. (a) What is the (linear) acceleration of the center point on the wheel? (b) What is the wheel's angular acceleration, with respect to the axis through its center?
7. Imagine a dumbbell consisting of two 1-kg point masses connected by a massless rod .4 m long. This dumbbell is then twirled about an axis through the middle of the rod and perpendicular to it, at a rate of 3 revolutions per second. (a) Calculate the moment of inertia of the dumbbell about this axis. (b) Calculate the kinetic energy of the dumbbell using the formula  $\frac{1}{2}I\omega^2$ . (c) What is the speed of each end of the dumbbell? (d) Calculate the kinetic energy again, using the formula  $\frac{1}{2}m|\vec{v}|^2$ .
8. Five solids with identical masses are shown in cross section below. The cross sections have equal widths at the widest parts and equal heights (but not necessarily equal thicknesses). (a) Which one has the greatest moment of inertia about an axis through

its center, perpendicular to the page? (b) Which has the smallest moment of inertia about such an axis?



9. The massive shield door at the neutron test facility at Lawrence Livermore National Laboratory has a mass of 44,000 kg and a width of 2.4 m. (a) From this information, make a *rough* estimate of the door's moment of inertia about the hinges (located at one edge as usual). (b) The actual moment of inertia is  $8.7 \times 10^4 \text{ kg}\cdot\text{m}^2$ . Neglecting friction, what steady force, applied at the outer edge of the door, is needed to move the door from rest through an angle of  $90^\circ$  in 30 seconds? (c) What force would be required to accomplish the same motion, if it were applied at the middle of the door?
10. A small solid marble, radius  $r$ , rolls down an inclined track and then around a "loop-de-loop" (radius  $R \gg r$ ) at the bottom of the hill. If the marble starts at rest from height  $h$ , what is the minimum value of  $h$  (in terms of  $R$ ) for the marble to remain in contact with the track all the way around the loop? conservation laws worksheet, and don't forget to include rotational kinetic energy. You should find that  $r$  cancels out of the final answer.)

## Study Guide

Gravity is a universal attractive force that acts between all objects, in proportion to their masses ( $m_1$  and  $m_2$ ) and inversely proportional to the distance ( $r$ ) between them:

$$|\vec{F}_g| = \frac{Gm_1m_2}{r^2}.$$

Here  $G$  is a universal constant whose measured value is  $6.67 \times 10^{-11} \text{ N}\cdot\text{m}^2/\text{kg}^2$ . You needn't memorize this formula or the value of  $G$ , but you should know that the gravitational force between ordinary objects is extremely small. To get a significant gravitational force, at least one of the two objects must be extremely massive (like the earth).

### Table of Analogies

| Linear Motion                            | Rotational Motion  |
|--|--|
| $t$                                      | $t$  |
| $x$                                      | $\theta$   |
| $v_x = \frac{\Delta x}{dt}$              | $\omega = \frac{\Delta \theta}{dt}$                      |
| $a_x = \frac{\Delta v_x}{dt}$            | $\alpha = \frac{\Delta \omega}{dt}$                      |
| <i>if</i> $a_x$ is constant,             | <i>if</i> $\alpha$ is constant,                          |
| $v_x = v_{x0} + a_x t$                   | $\omega = \omega_0 + \alpha t$                           |
| $x = x_0 + v_{x0}t + \frac{1}{2}a_x t^2$ | $\theta = \theta_0 + \omega_0 t + \frac{1}{2}\alpha t^2$ |
| $m$                                      | $I$  |
| $F_x$                                    | $\tau$   |
| net $F_x = ma_x$                         | net $\tau = I\alpha$                                     |
| $\text{KE} = \frac{1}{2}mv_x^2$          | $\text{KE} = \frac{1}{2}I\omega^2$                       |
| $p_x = mv_x$                             | $L = I\omega$  |
| <i>if</i> net external force = 0,        | <i>if</i> net external torque = 0,                       |
| net $p_x$ doesn't change                 | net $L$ doesn't change                                   |

Relations between linear and rotational quantities:

$$\omega = \frac{v_t}{r} \quad \alpha = \frac{a_t}{r} \quad I = \sum_i m_i r_i^2 \quad \tau = r|\vec{F}| \sin \phi \quad L = \sum_i m_i r_i |\vec{v}_i| \sin \phi_i$$