

### Problem Set 13

(due Friday, April 16)

1. Most of you were probably taught that normal human body temperature is  $98.6^{\circ}\text{F}$ . (Actually this number has too many insignificant figures, but never mind.) Convert this temperature to Celsius. Now suppose you are told to call the doctor if you child's fever reaches  $103^{\circ}\text{F}$ , but you have only a Celsius thermometer. At what point should you call the doctor?
2. What is normal human body temperature on the kelvin scale?
3. In 1964, the temperature in the Siberian village of Oymyakon reached a value of  $-71^{\circ}\text{C}$ . The highest officially recorded temperature in the continental U.S. was  $134^{\circ}\text{F}$ , in Death Valley, CA. Convert each of these temperatures to the Celsius scale, and also to the kelvin scale.
4. The Pyrex glass mirror in the telescope at the Mt. Palomar Observatory has a diameter of 200 in. Suppose that the temperature of the mirror ranges from  $-10^{\circ}\text{C}$  to  $50^{\circ}\text{C}$ . What is the maximum change in the diameter of the mirror?
5. Global warming will produce rising sea levels partly due to melting ice caps, but also due to the expansion of water as average ocean temperatures rise. To get a very rough estimate of what to expect in coming decades, calculate the change in the height of a column of water that is 1.0 km high, due to a temperature increase of  $1^{\circ}\text{C}$ .
6. Gold has a molar mass of 197 g/mol. (a) How many moles of gold are in a sample with a mass of 2.5 g? (b) How many atoms are in the sample?
7. Estimate the number of air molecules in your living room (within a factor of 2 or so).
8. Suppose that your car's tires are inflated to a gauge pressure of 32 psi on a winter day when the temperature is  $-10^{\circ}\text{C}$ . A few days later, spring arrives and the temperature climbs to  $20^{\circ}\text{C}$ . Assuming that the tires do not allow the air to expand in volume, what is gauge pressure at the new temperature? (Hint: Use the ideal gas law, and remember to distinguish between gauge pressure and absolute pressure.)
9. Rooms A and B are the same size, and are connected by an open door. Room A, however, is warmer (since its windows face the sun). Which room contains the greater mass of air? (Hint: use the ideal gas law.)
10. Estimate the total translational kinetic energy of all the air molecules in your living room.
11. The average speed of the nitrogen molecules in the air you're breathing is about 500 m/s. Air also contains trace amounts of krypton, a rather heavy gas. What is the rms speed of the krypton molecules at room temperature?
12. While camping in the mountains, you notice that your noodles boil at a temperature of  $90^{\circ}\text{C}$ . By referring to the table on page 312 of your textbook (saturation vapor pressure of water), determine what the atmospheric pressure is at your location, as a percentage of its sea-level value.
13. Suppose that the temperature is  $20^{\circ}\text{C}$  and the relative humidity is 50%. What percentage of the air molecules around you are actually molecules of water vapor?

## Study Guide

The **Celsius** temperature scale is defined so that the freezing point of water is  $0^{\circ}\text{C}$  and the boiling point (at 1 atm pressure) is  $100^{\circ}\text{C}$ . Experiments indicate that **absolute zero** (the lowest possible temperature) is then at  $-273^{\circ}\text{C}$ . A temperature in kelvin is equal to the same temperature in  $^{\circ}\text{C}$  plus 273, so absolute zero is 0 K. Room temperature is roughly 300 K.

I'll give you the equations that define the thermal expansion coefficients,

$$\frac{\Delta L}{L} = \alpha \Delta T, \quad \frac{\Delta V}{V} = \beta \Delta T.$$

You should know how to use these equations. You needn't memorize any numerical values of  $\alpha$  or  $\beta$ .

The **ideal gas law**,

$$PV = Nk_B T = nRT,$$

is quite accurate for all gases except at high density. Here  $P$  is absolute pressure,  $V$  is volume, and  $T$  is temperature *in kelvins*. Capital  $N$  stands for the number of *molecules* in the gas, while lower-case  $n$  stands for the number of *moles*. These are related by

$$N = n \cdot N_A,$$

where  $N_A$  is **Avogadro's number**,  $6.0 \times 10^{23}$ . (This is approximately the number of hydrogen atoms in one gram of hydrogen.) The experimental value of the constant  $R$  is 8.3 J/K, and from this you can show that Boltzmann's constant is

$$k_B = \frac{R}{N_A} = 1.38 \times 10^{-23} \text{ J/K}.$$

Please memorize the values of  $N_A$  and  $R$ ; from these you can calculate  $k_B$ .

At the sub-microscopic scale, a gas is made of molecules that bounce around at high speeds, obeying Newton's laws of motion (with no friction). The pressure of the gas is caused by collisions with the walls of the container. From Newton's laws and the ideal gas law it follows that the average translational kinetic energy of a gas molecule is

$$\overline{\text{KE}} = \frac{3}{2} k_B T.$$

Thus, Boltzmann's constant is essentially a *conversion factor* between temperature and molecular energy. If you substitute the standard formula for KE, you can solve for the "rms" speed of the molecules as a function of  $T$ . Many types of molecules can also store rotational and vibrational energy, so there is no universal relation between the temperature of a system and its *total* energy.

If the molecules in a system are moving slowly enough, attractive forces will cause them condense into a liquid, in which the molecules are always "touching" their neighbors. Removing still more energy causes the molecules to settle into a regular arrangement—a solid crystal. The three states of matter (gas, liquid, and solid) are called **phases**. Phase changes can be caused by changes in either temperature or pressure.

You should have a basic understanding of the concepts of vapor pressure, relative humidity, and phase diagrams.