

**Problem Set 10**  
(due Tuesday, March 30)

1. At typical capillary has a radius of  $2\ \mu\text{m}$ , and blood flows through it at a rate of  $3.8 \times 10^{-9}\ \text{cm}^3/\text{s}$ . (a) What is the speed of the blood flowing through the capillary? (The small speed allows time for diffusion of materials to and from the blood.) (b) Assuming that all the blood in the body passes through capillaries, how many of them must there be to carry a total flow of  $90\ \text{cm}^3/\text{s}$ ? (According to your textbook, your answer will be somewhat of an overestimate.)
2. Water flows through a garden hose at a speed of  $2\ \text{m/s}$ ; the internal diameter of the hose is  $1.6\ \text{cm}$ . (a) What is the flow rate in liters per second? In gallons per minute? (b) The water's velocity at it leaves the hose's nozzle is  $15\ \text{m/s}$ . What is the nozzle's inside diameter?
3. Suppose you were to drill a hole near the base of Hoover Dam,  $700\ \text{ft}$  below the water level of Lake Mead. At approximately what speed would the water come rushing out of the hole?
4. The windows in an office building each measure  $4\ \text{m}$  by  $5\ \text{m}$ . On a stormy day, air is blowing at  $30\ \text{m/s}$  past a window on the top floor. Estimate the net force on the window. (Take the density of the air to be  $1.23\ \text{kg}/\text{m}^3$ .)
5. Look up the definition of the coefficient of viscosity in your textbook, explain this definition in your own words with a picture, and use the definition to determine the units of the coefficient of viscosity (usually denoted by  $\eta$ , the Greek letter "eta").
6. Imagine water flowing through a pipe, perhaps in your home's plumbing system. Determine how each of the following changes would affect the water's flow rate, and in each case, explain in your own words *why* the flow rate would change in this way.
  - (a) The length of the pipe is doubled (without changing the pressure at either end).
  - (b) The difference in pressure between the two ends is doubled.
  - (c) The water's temperature increases from  $0^\circ\text{C}$  to  $40^\circ\text{C}$ , causing its viscosity to change.
  - (d) The pipe is replaced with a new one whose diameter is twice as large.
7. The alternating electrical voltage supplied by power outlets in the U.S. oscillates (from positive to negative and back again) with a frequency of  $60\ \text{Hz}$ . What is the period of oscillation?
8. A tuning fork vibrates with a period of  $2.5 \times 10^{-3}$  seconds. What is its frequency of vibration?
9. To "weigh" themselves in space, astronauts use a device that is essentially a chair on a spring. For the device used on Skylab (the first U.S. space station), the spring constant was  $600\ \text{N/m}$ , and the period of oscillation of the empty chair was  $0.9$  seconds. (a) What was the mass of the chair? (b) Suppose that, with an astronaut sitting in the chair, the period of oscillation was  $2.1$  seconds. What was the mass of the astronaut?
10. Suppose that the astronaut in the previous problem is oscillating with an amplitude of  $5\ \text{cm}$  (measured in either direction from the equilibrium position). (a) What is the maximum energy stored in the spring? (b) What is the astronaut's maximum speed during the motion? (Hint: Use energy conservation.)

11. The formula for the period of a simple pendulum (in the limit of small amplitude) is  $T = 2\pi\sqrt{L/g}$ . Explain how you could “guess” this formula (aside from the factor of  $2\pi$ ) using unit analysis.
12. You wish to design a pendulum clock so that it will “tick” exactly once per second. This means that the period of the pendulum should be exactly 2 seconds (since it ticks when swinging in each direction). How long should the pendulum be?
13. After carefully adjusting your pendulum clock to tick at the correct rate in your basement, you take the clock to a mountain top and notice that it runs too slow—losing one second per hour. How does the local gravitational constant on the mountain top compare to its value in your basement? (That is, is it more or less, and by what percentage?)

## Study Guide

### Fluid Dynamics

The flow rate of a fluid is the volume per unit time passing a given point. You should be able to relate flow rate to the speed of the fluid and the cross-sectional area of the pipe or channel.

For an incompressible fluid in a closed pipe, the flow rate is the same at all points. This implies

$$A_1v_1 = A_2v_2,$$

where  $A$  is the cross-sectional area and  $v$  is the speed. (This is the “squirt gun” principle.)

The work-energy relation, applied to an incompressible, frictionless fluid, implies that

$$P + \rho gh + \frac{1}{2}\rho v^2$$

is the same at all points. This is “Bernoulli’s principle.” (The  $P$  term is related to work, the  $\rho gh$  to gravitational energy, and the  $\frac{1}{2}\rho v^2$  to kinetic energy.) You should understand (and be able to give examples of) the three special cases of this principle: when  $P$  doesn’t vary; when  $h$  doesn’t vary; and when  $v$  doesn’t vary. You needn’t memorize this expression.

You should have a qualitative understanding of the concept of viscosity, and how it affects the flow rate of real fluids in real pipes.

### Oscillations

You should understand the terms *frequency* and *period*, and how they relate to each other ( $f = 1/T$ ). You should know that “Hz” (hertz) is another name for the unit of frequency,  $\text{s}^{-1}$  (or oscillations per second).

You needn’t memorize the formulas for the period of oscillation of a mass-on-spring or simple pendulum, but you should be able to figure out these formulas (aside from the factors of  $2\pi$ ) by analyzing units, and you should be able to apply these formulas to various situations.