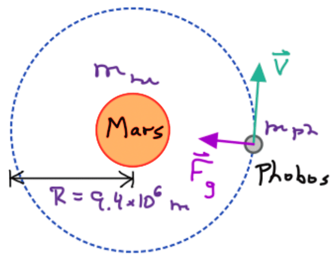
Example Problem and Solution

To illustrate what a good problem solution looks like, here is a problem from Physics 2210:

- The planet Mars has a satellite (moon), called Phobos, which travels in an approximately circular orbit of radius 9.4×10^6 m with a period of 7 hours, 39 minutes. From this information and Newton's law of gravity, determine the mass of Mars.

Solution:



This is a constrained motion problem (circular motion), so use Newton's 2^{nd} law:

$$\sum \vec{F}_{\text{on ph.}} = m_{ph.} \vec{a}_{ph.}$$

($T = 7$ hr $+ 39$ min $= 459$ min $= 27{,}540$ s) $\longrightarrow \vec{F}_g = m_{ph} \vec{a}_{ph.}$ (just one force).

Take magnitude of both sides:

$$|\vec{F}_g| = m_{ph} |\vec{a}| = m_{ph} \cdot \frac{|\vec{v}|^2}{R} \quad \text{for circular motion}$$

$$\rightarrow \frac{G m_m \cancel{m_{ph}}}{R^{\cancel{2}}} = \frac{\cancel{m_{ph}} |\vec{v}|^2}{\cancel{R}} \rightarrow m_m = \frac{R}{G} |\vec{v}|^2$$

$$\rightarrow m_m = \frac{R}{G} \cdot \left(\frac{2\pi R}{T} \right)^2 = \frac{4\pi^2 R^3}{G T^2} \quad \text{(Kepler's } 3^{rd} \text{ law)}$$

$$= \frac{4\pi^2 (9.4 \times 10^6\, \cancel{m})^3}{(6.67 \times 10^{-11}\, \frac{N \cdot \cancel{m^2}}{kg^{\cancel{2}}})(27540\, \cancel{s})^2} = \underline{6.5 \times 10^{23}\, kg}, \text{ about}$$

$N = \cancel{\frac{kg \cdot m}{s^2}}$

11% of earth's mass.

Things to notice:

- The well-labeled *picture* that defines the symbols used.
- English *words* that state what principles apply (Newton's 2nd law and circular motion), guide the reader through the steps, and add insightful comments on the results (Kepler's 3rd law and the comparison to earth's mass).
- The use of *arrows* to indicate where one equation implies the next.
- All numbers include *units*, allowing the author to check (not assume) that the final answer is in kilograms.
- The final answer is *rounded* to two significant digits, roughly consistent with the precision of the given orbital radius.