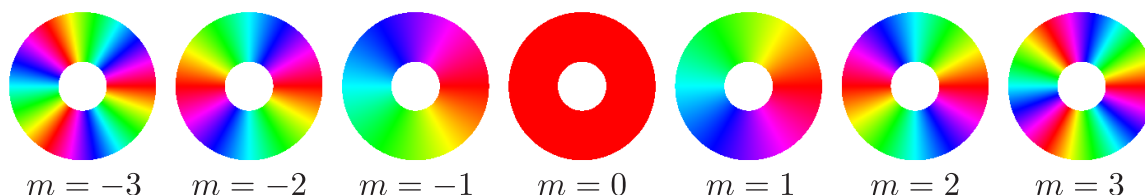
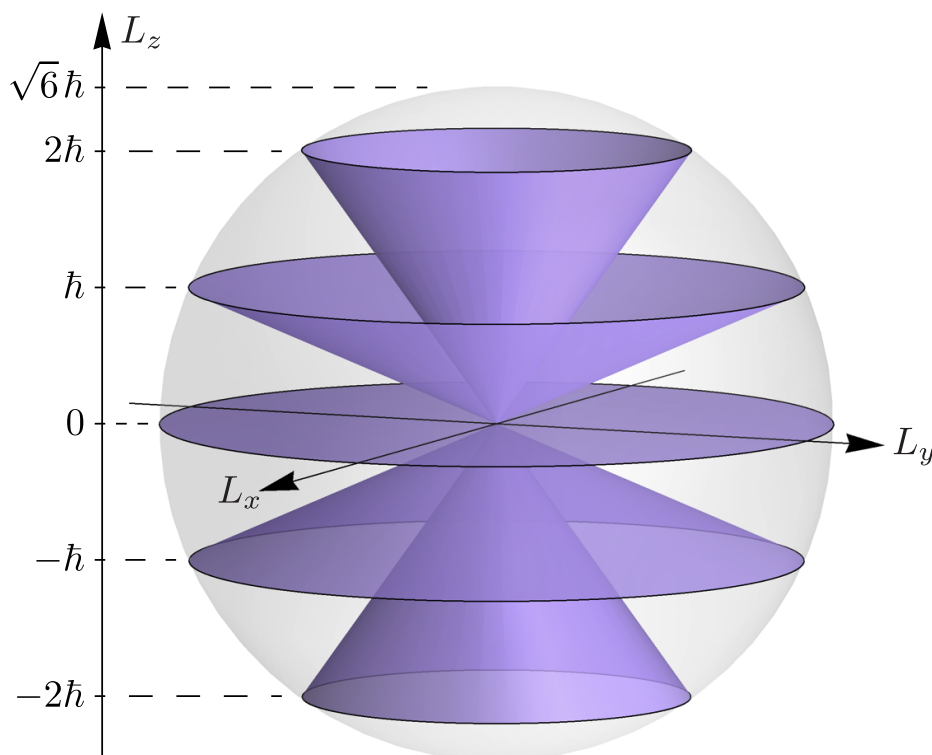


Picturing Angular Momentum

In two dimensions, a wavefunction of definite angular momentum has the angular dependence $e^{im\phi}$, where ϕ is the angle in polar coordinates and m can be any integer. The corresponding value of the angular momentum is $m\hbar$. Here is what some of these functions look like when we represent the complex phase using color hues in the usual way:



In three dimensions, only one component of the angular momentum vector $\vec{L}$ can be well defined. We conventionally take this to be the z component. Meanwhile the magnitude $|\vec{L}|$ can also be well defined, but is always larger than the magnitude of L_z . This means that when both L_z and $|\vec{L}|$ are well defined we can calculate the value of $L_x^2 + L_y^2$ even though we can't specify either L_x or L_y separately. Here is a “cone diagram” for visualizing the five allowed L_z states when the quantum number ℓ equals 2, so that $|\vec{L}| = \sqrt{6}\hbar$:



Here is a visualization of the wavefunctions that have both definite $|\vec{L}|$ and definite L_z . Each wavefunction depends on two angles: a polar angle θ (measured away from the positive z axis) and an azimuthal angle ϕ (measured going around the xy plane). Here I've plotted each wavefunction on the surface of a sphere, with the z axis pointing upward. The magnitude of the wavefunction determines the brightness, with black indicating a value of zero (a node). Note that the ϕ dependence of each wavefunction is simply $e^{im\phi}$, as in two dimensions. The θ dependence can be written in terms of sines and cosines. These functions of θ and ϕ are called *spherical harmonics*, and if you want to learn more about them you can look them up on the internet or in Mathematica.

