Electromagnetic Theory
Fall 2019

Problem Set 8

(due Monday, November 4, 4:00 pm)

1. (a) A square wire loop lies in the same plane as a long, straight wire (see Fig. 5.24 in your text). The square has width a and its nearest side is parallel to the long wire and a distance s away from it. Both the square and the long wire carry the same current I, with the current in the near side of the square running antiparallel to that in the long wire. Find the force exerted by the long wire on the square. (b) Repeat this problem but with the square replaced by an equilateral triangle, each side having length a. The near side is still parallel to the long wire and has a current running opposite to that in the long wire. This time you will have to set up an integral to calculate the force on the diagonal sides. (If you find this difficult, please put this calculation aside and come back to it after you've finished the rest of this assignment.) (c) Using any reasonable values for I, a, and s, evaluate each of your results numerically and check that the results are reasonable and have correct units.

2. A steady current I flows down a long cylindrical wire of radius a. Find the magnetic field, both inside and outside the wire, in each of the following cases: (a) the current is distributed uniformly over the outside surface of the wire; (b) the current is distributed uniformly throughout the volume of the wire; (c) the current is distributed in such a way that J is directly proportional to the distance s from the axis of the wire.

3. In calculating the current enclosed by a given Amperian loop, you need to evaluate an integral of the form $\int \mathbf{J}\cdot d\mathbf{a}$, over the surface bounded by the loop. The trouble is, there are infinitely many different surfaces that share the same boundary line. Which one are you supposed to use? (This is a trick question, but it still has an unambiguously correct answer. Be sure to justify your answer!)

4. Find the vector potential around an infinitely long, straight wire carrying a current I, in terms of the distance s from the wire. Hints: I started by guessing the direction of $\mathbf{A}$, and using symmetry to argue that $\mathbf{A}$ can't depend on z (the direction parallel to the wire). Then it was easy to show that $\nabla \cdot \mathbf{A} = \mathbf{0}$, and only a little more difficult to use the formula for the curl in cylindrical coordinates to relate the unknown part of the formula for $\mathbf{A}$ to the known formula for $\mathbf{B}$.

5. In this problem you will use Mathematica to plot the magnetic field and magnetic vector potential of a circular current-carrying loop of wire. (You may wish to warm up by reviewing the earlier problem in which you calculated the *electric* field and the *electric* potential of a *charged* loop.) Please use units in which $\mu_0 I/(4\pi) = 1$ and the radius of the loop equals 1. Let the loop lie in the xy plane; then it is sufficient (by symmetry) to plot the field in the xz plane. First prove that $A_z = 0$, so there are only two components of the vector potential to calculate. Then carefully devise explicit formulas for the x and y components of the vector potential, breaking up the loop into about 50 segments and summing over the contributions of these segments (use the angle ϕ as the variable to sum over). These formulas should be valid at *any* point in 3D space; don't assume $y = 0$ yet (as I incorrectly did the first time I worked

this problem!). Please write these explicit formulas for A_x and A_y *by hand* before attempting to do anything with Mathematica. Next, use Mathematica to calculate the x and z components of $\mathbf{B}$ by taking the appropriate derivatives of A_x and A_y. (There is no need to calculate B_y.) *Now* you can set $y = 0$ and note that A_x is zero there, so it makes sense to make a contour plot of A_y and overlay a vector plot of $\mathbf{B}$. So that's what you should do, adjusting the plotting parameters until the plot shows the field's behavior reasonably well.

6. A circular loop of wire, with radius R, lies in the xy plane (centered at the origin) and carries a current I running counterclockwise as viewed from the positive z axis. (a) What is its magnetic dipole moment? (b) What is the (approximate) magnetic field at points far from the origin? (c) Show that, for points on the z axis, your answer is consistent with the *exact* field (as calculated in your textbook) when $z \gg R$.