Electromagnetic Theory
Fall 2019

Problem Set 6

(due Wednesday October 16, 12:30 pm)

1. For the physical dipole discussed in class (and in Example 3.10), expand $1/\mathscr{r}_\pm$ to order $(d/r)^3$, and use this to determine the quadrupole and octopole terms in the potential. Be sure not to discard the $(d/r)^2$ terms from the law of cosines!

2. A hydrogen atom (whose effective radius is the Bohr radius, about half an angstrom) is situated between two metal plates 1 mm apart, which are connected to opposite terminals of a 500 V battery. The atom is polarized by this field, so that the center of the electron charge distribution is displaced a distance d from the nucleus. (a) Look up the value of α in your textbook, then calculate d and estimate, roughly, what fraction of the atomic radius this separation amounts to. Is it a lot or a little? (b) Make a rough estimate of the voltage you would need, in this apparatus, to ionize the atom.

3. A point charge q is located at a large distance r from a neutral atom of polarizability α. Find the force of attraction between them. (You may either calculate the force exerted by the atom on the point charge, or the force exerted by the point charge on the atom. Do whichever is easier!)

4. A dipole $\mathbf{p}$ is placed in an externally created electric field $\mathbf{E}$. Because the field exerts a torque, the dipole's electrostatic potential energy depends on its orientation. Show that this energy can be written $U = -\mathbf{p} \cdot \mathbf{E}$, and interpret this result in words.

5. A cylinder of radius a and length L carries a "frozen-in" uniform polarization $\mathbf{P}$, parallel to its axis. Find the bound charge, and clearly sketch the electric field, both inside and outside the cylinder, assuming that $L \approx 2a$. Make separate sketches showing $\mathbf{P}$ and $\mathbf{D}$ in and around this cylinder. Explain briefly how you determined each feature of your sketches. (This object is called a *bar electret*; it is the electrical analog of a bar magnet. Bar electrets are much less common than bar magnets, because only rare materials can hold a permanent electric polarization. The most famous example is barium titanate.)

6. Suppose you have enough linear dielectric material, of dielectric constant ϵ_r, to *half*-fill a parallel-plate capacitor. Two ways to distribute this material would be (a) to spread it over the full area, with only half the thickness needed to span the plates; or (b) to make it fill the full thickness, but spread over only half the area. By what fraction is the capacitance of this capacitor increased in each of these cases? For a given potential difference V between the plates, find $\mathbf{E}$, $\mathbf{D}$, and $\mathbf{P}$ in each region, and the free and bound charge on all surfaces, for both cases.

7. **Writing exercise.** In Problem Set 4, Problem 6(c), you calculated the electrostatic energy of a uniform sphere of charge using the formula for energy in terms of the electric field,
$$W = \frac{\epsilon_0}{2} \int |\mathbf{E}|^2 \, d\tau.$$

You should have obtained the answer

$$W = \frac{3}{5}\frac{1}{4\pi\epsilon_0}\frac{Q^2}{R},$$

where Q is the total charge of the sphere and R is its radius. Now please write a formal presentation of that calculation, with everything organized into sentences and paragraphs. I suggest that you type your writeup, but if you're not comfortable with typing equations then you may write it neatly by hand. Please refer to the accompanying page for tips on correctly incorporating mathematics into English writing. Also please embellish your writeup with some introductory motivation and some insightful concluding remarks, to make it into a complete essay that is interesting to read. Your essay need not include the derivations of the formulas for the electric field inside and outside the sphere, but you should present these formulas clearly before you use them.