Electromagnetic Theory
Fall 2019

Problem Set 4

(due Monday, September 30, 4:00 pm)

1. Consider two possible formulas for electrostatic fields: (a) $\mathbf{E} = k[xy\hat{\mathbf{x}} + 2yz\hat{\mathbf{y}} + 3xz\hat{\mathbf{z}}]$; or (b) $\mathbf{E} = k[y^2\hat{\mathbf{x}} + (2xy + z^2)\hat{\mathbf{y}} + 2yz\hat{\mathbf{z}}]$. One of these formulas is impossible. Which one is that? For the *possible* one, find the electrostatic potential by carrying out a line integral, using the origin as your reference point. Check your answer by computing ∇V. (Hint: You must select a specific path to integrate along, and state clearly what path you have chosen. It doesn't matter what path you choose, since the answer is path-independent, but you simply cannot integrate unless you choose a specific path for the integral.)

2. Find the potential inside and outside a uniformly charged solid sphere of radius R and total charge q, starting from the formulas for the electric field that you already found in a previous problem set. Use infinity as your reference point. Compute the gradient of V in each region, and check that it yields the correct electric field. Sketch a graph of $V(r)$, with V on the vertical axis and r on the horizontal axis. Be sure to check your answer to this problem, because you will use it in Problem 6(b) below.

3. Redo Problem 3 of Problem Set 3, but this time plot both the potential (using `ContourPlot`) and the electric field (using `VectorPlot`) for each of the three field configurations: (a) a point charge, (b) a dipole, and (c) a straight, uniformly charged line segment modeled as n point charges. (Set $n = 100$ or more for the plot that you turn in.) Use `Show` to combine the potential and field plots into a single plot for each configuration, and choose colors to produce reasonably good contrast. For the dipole, please use a color scheme that assigns white to $V = 0$. You may also want to use the `PlotRange` option to determine the range of values in the contour plots of the potential. Note that if you had trouble finding the correct formulas for E_x and E_y when you did the earlier problem, you can obtain them a little more easily if you calculate the potential first and then take its partial derivatives, e.g., `ex = -D[v,x]`.

4. Use Mathematica (as in the previous problem) to calculate and plot the potential and electric field around a uniform circular ring of charge, modeling it as a collection of n equally spaced point charges as in Problem 3(c). Put the ring in the xy (horizontal) plane, centered at the origin, with radius 1 in natural units. Your plot should show a slice along the xz plane, that is, perpendicular to the ring. It is easiest to calculate the potential first and then take its partial derivatives to obtain the electric field components.

5. (a) Three point charges are located at the corners of a square of width a: $-q$ at each of two diagonally opposite corners, and $+q$ at one of the remaining corners. The fourth corner is initially empty, but now we wish to bring in another charge $+q$ from far away and place it there. How much work is required to do this, in terms of q and a? (b)

How much work is required to assemble the entire final configuration of four charges, starting with all of them very far apart?

6. This problem is all about the electrostatic energy of a uniformly charged solid sphere.

(a) Use dimensional analysis to find a formula for the electrostatic energy of a uniformly charged solid sphere of radius R and total charge q. That is, find the unique combination of R, q, and ϵ_0 that has units of energy (and explain how you can tell it's unique). Your formula will have an undetermined unitless constant factor, which you will determine in the following parts.

(b) Find the formula for the total electrostatic energy of a uniformly charged sphere of radius R and total charge q, starting with the general formula $\frac{1}{2}\int \rho V \, d\tau$.

(c) Find the formula for the total electrostatic energy of a uniformly charged sphere of radius R and total charge q, starting with the general formula $\frac{\epsilon_0}{2}\int |\mathbf{E}|^2 \, d\tau$.

(d) Find the formula for the total electrostatic energy of a uniformly charged sphere of radius R and total charge q, by starting "from scratch" and setting up an integral for the total work performed when you assemble the sphere out of successive concentric shells. (Be sure to draw a good picture!)

(e) In rough approximation, the nuclei of heavy atoms can be described, so far as their electrical structure is concerned, as balls of matter with the constant volume charge density 1.3×10^{19} coulombs per cubic centimeter. If a uranium nucleus with total charge $92e$ (where e is the fundamental unit of charge) splits into two nuclei of equal charge and radius, which then separate far apart, what is the change in electrical energy, expressed first in joules and then in MeV? (To calculate the change, subtract the final electrostatic energy from the initial electrostatic energy.) If a kilogram of uranium underwent such a transformation, how many joules of electric energy would be released? (Note that this problem concerns only the *electrostatic* energy of the nuclei; there is also energy associated with the strong *nuclear* force, but that is much harder to calculate.)