

Problem Set 10

(due Friday, November 22, 4:00 pm)

- Two concentric metal spherical shells, of radii a and b , respectively (with $a < b$), are separated by weakly conducting material of uniform conductivity σ .
 - If they are maintained at a potential difference ΔV , what current flows from one to the other (in terms of a , b , and σ)?
 - What is the resistance between the shells?
 - Notice that if $b \gg a$, the outer radius (b) is irrelevant. How do you account for that? Exploit this observation to determine the current flowing between two metal spheres, each of radius a , immersed in the deep sea and held quite far apart, if the potential difference between them is ΔV . (This arrangement can be used to measure the conductivity of sea water.)
- A capacitor C has been charged up to a potential V_0 . At time $t = 0$ it is connected to a resistance R and begins to discharge. (Please express all answers to this problem in terms of C , V_0 , R , and t .)
 - Find explicit formulas for the charge on the capacitor and the current through the resistor as functions of time. (You'll need to set up and solve a simple differential equation.)
 - What is the initial energy stored in the capacitor? By integrating the expression $I^2 R$ for the power through the resistor, confirm that the "heat delivered" to the resistor is equal to the energy lost by the capacitor.
 - Now imagine *charging up* the capacitor, by connecting it and the resistor in series to a battery of voltage V_0 at $t = 0$. Again, determine $Q(t)$ and $I(t)$.
 - By integrating the product IV_0 , with the function you just found for I , find the total energy output of the battery. Integrate $I^2 R$ again, to find the "heat delivered" to the resistor in this case. What fraction of the work done by the battery shows up as energy in the capacitor?
- A long rectangular circuit loop is placed so that one end of it is inside a charged parallel-plate capacitor, oriented parallel to the capacitor's electric field. The other end of the loop is outside the capacitor and very far away. What is the emf produced in this circuit by the capacitor's electric field? Can you use this device to create a perpetual motion machine? Explain carefully.
- A square wire loop measuring $a \times a$ is mounted on a rotating shaft that runs down its middle (in the same plane as the loop, splitting it in half), and is then rotated at constant angular velocity ω . A uniform magnetic field \mathbf{B} points perpendicular to the shaft, so the loop is alternately parallel and perpendicular to the field as it spins. Find the emf in this loop, as a function of time, in two different ways: (a) using the Lorentz force law and the definition of emf; and (b) using the flux rule.

5. A long solenoid with radius a and n turns per unit length carries a time-dependent current $I(t)$ in the $\hat{\phi}$ direction. Find the electric field (magnitude and direction) at a distance s from the axis, both inside and outside the solenoid, in the quasistatic approximation. (Your answer will depend on the unknown function dI/dt .) Sketch a graph of $|\mathbf{E}|$ vs. s . Also make a separate sketch of the electric field vectors (using arrows) as viewed from one end of the solenoid, for a particular choice of the sign of dI/dt (which you should clearly specify).
6. A square loop, $a \times a$, with resistance R , lies in a horizontal plane along with an infinitely long, straight wire that carries a current I (see Fig. 7.29 in your textbook). Two sides of the square are parallel to the long wire, and the nearest side is a distance s_0 away from the wire. Now someone cuts the wire, so the current in it suddenly drops to zero. The changing magnetic field induces a current to flow in the square loop. What is the direction of this induced current? Find the total charge that flows past a given point in the square loop, in terms of a , I , R , and s_0 .
7. Two circular loops of wire are oriented horizontally, one above the other, centered on a common axis, with their centers separated by a distance z . The upper loop is very small (compared to z), with radius a , while the lower loop is larger, with radius b .
 - (a) Suppose current I flows in the big loop. Find the magnetic flux through the small loop. (The small loop is so small that you may treat the field of the big loop as uniform.)
 - (b) Suppose current I flows in the small loop. Find the magnetic flux through the big loop. (The small loop is so small that you may treat it as an ideal dipole.)
 - (c) Find the mutual inductances, and confirm that $M_{12} = M_{21}$.
8. Calculate the energy stored in a long cylindrical solenoid with length ℓ , radius R , current I , and n turns per unit length, in three different ways: (a) using the formula $\frac{1}{2}LI^2$ for the energy stored in an inductor; (b) using the formula $\frac{1}{2} \oint (\mathbf{A} \cdot \mathbf{I}) dl$ in terms of the current and the vector potential; and (c) using the formula $(1/2\mu_0) \int B^2 d\tau$ in terms of the magnetic field. You may look up the formulas for L , \mathbf{A} , and \mathbf{B} in your textbook and/or class notes.

If by anything I have here written I may assist any student in understanding Faraday's modes of thought and expression, I shall regard it as the accomplishment of one of my principal aims—to communicate to others the same delight which I have found myself in reading Faraday's Researches.

—James Clerk Maxwell, *A Treatise on Electricity and Magnetism*