From Clouds to Cosmology: New and Old Applications of Thermal Physics

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Cumulus Clouds

A cumulus cloud is the result of convection plus condensation. Under what conditions will atmospheric convection occur? Assuming that a rising parcel of air expands adiabatically, its temperature will drop as it rises to regions of lower pressure. In order for it to keep rising, its temperature must remain higher than that of the surrounding air. Quantitatively, it's not hard to show that, for any ideal gas undergoing adiabatic expansion, $dT/dP = [(\gamma-1)/\gamma]T/P$. If the parcel is just barely unstable to convection, then hydrostatic equilibrium tells us that $dP/dz = -(mg/kT)P$. Combining these equations gives us the critical temperature gradient for convection, called the "dry adiabatic lapse rate":

$$\left|\frac{dT}{dz}\right| = \left(\frac{\gamma-1}{\gamma}\right)\frac{mg}{k} \approx 10^\circ\text{C/km} \qquad \text{for Earth's atmosphere.}$$

This is also the rate at which the parcel's temperature drops as it rises. To determine when condensation occurs, all we need is the initial value of the relative humidity and the phase diagram for water (which is virtually unchanged by the presence of air). For example, at 25°C and 50% relative humidity, the partial pressure of water vapor in the air is about 0.016 bar. This is equal to the vapor pressure of water at 14°C, and a rising parcel will drop to this temperature at an altitude of 1.1 km. Adjust for the fact that the partial pressure drops (along with the total pressure) with altitude, and you find that condensation is postponed slightly, until an altitude of 1.4 km.

Reference: Craig Bohren and Bruce Albrecht, *Atmospheric Thermodynamics* (Oxford University Press, New York, 1998).

Contraction of Muscles

A muscle is like a fuel cell, operating on the reaction

$$\text{glucose} + 6O_2 \longrightarrow 6CO_2 + 6H_2O.$$

The net energy released per mole during this reaction is $|\Delta H| = 2810$ kJ, while the maximum energy available as *work* is slightly higher, $|\Delta G| = 2880$ kJ. (The difference is due to the fact that the products have more entropy than the reactants, and therefore the system can absorb heat as the reaction occurs without violating the second law.) An intermediate step in this reaction is the production of ATP (adenosine triphosphate); for each glucose molecule we get 38 molecules of ATP. The muscle is then powered by the breakdown of ATP to ADP plus phosphate, catalyzed by an enzyme called myosin that makes up part of the muscle filaments. Each time a myosin catalyzes this reaction, it pulls on an adjacent filament with an average force of about 4 piconewtons over a distance of about 11 nm. The work performed during the reaction is therefore 4.4×10^{-20} J. Each glucose molecule gives us 38 ATP's, so the total work we can get out of a mole of glucose is

$$(4.4 \times 10^{-20}\ \text{J}) \cdot (38) \cdot (6 \times 10^{23}) = 1000\ \text{kJ}.$$

That's a 35% efficiency, compared to the maximum allowed by the laws of thermodynamics.

Reference: Lubert Stryer, *Biochemistry*, fourth edition (W. H. Freeman, New York, 1995).

Carbon

Everyone knows that elemental carbon has two common forms, diamond and graphite. At earth's surface the stable form is graphite, as evidenced by the fact that ΔG for converting a mole of graphite to diamond is positive, 2.9 kJ. But as you increase the pressure, G for either phase increases in proportion to its volume: $(\partial G/\partial P)_T = V$. Since graphite has the greater molar volume, its G value increases faster with pressure, eventually overtaking that of diamond at a pressure of about 15,000 atmospheres. At higher pressures, the entropy gained by the environment (because it gained space, even though it lost energy) outweighs the entropy lost by the system as it converts from graphite to diamond.

We now have one point to plot on the graphite-diamond phase boundary. To complete the phase diagram, we can use the Clausius-Clapeyron relation $dP/dT = \Delta S/\Delta V$. The entropy difference ΔS changes somewhat with temperature, but can be calculated from heat capacity data. Thus, we can predict the entire graphite-diamond phase diagram with fair accuracy, entirely from data that can be obtained (at least in principle) at low pressures.

References: Keith G. Cox, "Kimberlite Pipes," *Scientific American* **238**, 120–132 (April, 1978); D. K. Nordstrom and J. L. Munoz, *Geochemical Thermodynamics*, second edition (Blackwell Scientific Publications, Palo Alto, 1994).

Computing

This application is mathematically very simple, but gets at the core of what entropy is all about. Imagine the process of erasing 1 gigabyte (2^{33} bits) of computer memory. Because the memory itself has $2^{2^{33}}$ possible initial states, whatever piece of hardware does the erasing has at least $2^{2^{33}}$ possible final states. That's an entropy of

$$S = k \ln 2^{2^{33}} = 2^{33} \cdot (\ln 2) \cdot k = (6 \times 10^9)k = 8 \times 10^{-14} \text{ J/K}.$$

Small, but nonzero. If the computer is able to perform this operation repeatedly, it must somehow dispose of this entropy as it goes along. If the environment is at room temperature, disposing of this entropy requires the output of heat in the amount

$$Q = T\,\Delta S = (300 \text{ K})(8 \times 10^{-14} \text{ J/K}) = 25 \text{ picojoules}.$$

Of course, today's computers expel much more heat than this as they process and discard information. But at the rate computer technology is progressing, who knows how long it will be before this thermodynamic limit becomes a significant constraint?

Reference: Hans Christian von Baeyer, *Maxwell's Demon* (Random House, New York, 1998).

Collapsed Stars

White dwarfs and neutron stars are degenerate Fermi gases held together by gravity; I won't say much more about them because they're now treated in most thermal physics textbooks. But what about black holes? Since you can't tell from looking at a black hole what kind of matter was used to create it, its entropy must be greater than that of any possible type of initial matter. The maximum-entropy initial matter would be a collection of long-wavelength, zero-mass particles such as photons, and the maximum wavelength of these photons would be roughly the final radius of the black hole, which must be of order GM/c^2 by dimensional analysis. Therefore we can estimate the entropy as

$$S_{\text{b.h.}} \sim Nk \sim \frac{Mc^2}{hc/\lambda} \cdot k \sim \frac{GM^2 k}{hc}.$$

The exact formula, derived by Hawking in 1973, also contains a factor of $8\pi^2$. For a one-solar-mass black hole this is an enormous entropy, roughly 10^{20} times that of an ordinary star of the same mass. By computing $dS/dM \propto dS/dU$, you can easily show that the temperature is $hc^3/16\pi^2 GMk$, which works out to only 60 nK for the sun's mass. And since a black hole is a blackbody if ever there was one, this temperature allows you to compute the rate at which it emits blackbody (Hawking) radiation.

References: Leonard Susskind, "Black Holes and the Information Paradox," *Scientific American* **276**, 52–57 (April, 1997); M. G. Bowler, *Lectures on Statistical Mechanics* (Pergamon Press, Oxford, 1982).

Cosmology

Of course the cosmic background radiation is an excellent example of blackbody radiation, with a spectrum given by the Planck formula,

$$u(\epsilon) \propto \frac{\epsilon^3}{e^{\epsilon/kT} - 1}.$$

But why stop with photons? To describe the cosmic neutrino background, just change the minus in the denominator to a plus (neutrinos are fermions), and be careful to count the three flavors, each with two polarization states. By computing the number of cosmic neutrinos at the present temperature of 1.95 K, you can even estimate what the neutrino mass would have to be in order for neutrinos to make a significant contribution to the dark matter. For even more fun, calculate the spectrum of electron-positron pairs in the very early universe; the only new twist here is the dispersion relation $\epsilon = \sqrt{p^2 + m^2}$ for massive particles, which forces you to do the integrals numerically. As the universe expanded and cooled, the neutrinos stopped interacting with the electrons and photons. Then, as the electrons disappeared (when kT dropped below mc^2), they "heated" the photons but not the neutrinos. It's not particularly hard to calculate the neutrino temperature as a function of photon temperature, and thus account for the present ratio $T_\gamma/T_\nu = (11/4)^{1/3}$. To figure out *when* all this happened, you need to add in some gravitational dynamics. Other thermodynamic calculations that aren't too hard include the proton/neutron ratio when the temperature was around 10^{11} K, and the temperature at which the universe first became transparent, when electrons and nuclei (mostly hydrogen) first combined to form stable atoms.

Reference: Steven Weinberg, *The First Three Minutes* (Basic Books, New York, 1977).