Homework #5

(27-1) a) For steady current, $i = \frac{\Delta q}{\Delta t}$, so

$$\Delta q = i\,\Delta t = (5.0A)(4\,min)\left(\frac{60\,sec}{1\,min}\right) = \boxed{1200\,C}$$

b) $\#\,electrons = \frac{\Delta q}{e} = \frac{1200\,C}{1.6\times10^{-19}\,C/electron} = \boxed{7.5\times10^{21}\,electrons}$

(27-2) For a uniform current, $i = JA = J(\pi r^2) = J(\pi \frac{d^2}{4})$ where d = wire diameter. So

$$d = \sqrt{\frac{4i}{J\pi}} = \sqrt{\frac{4(0.5A)}{(440\,A/cm^2)\left(\frac{100\,cm}{1m}\right)^2\pi}} = \boxed{3.80\times10^{-4}\,m}$$

(27-3) a) $i = \frac{V}{R} = \frac{23\,V}{15\times10^{-3}} = \boxed{1530\,A}$ (a huge current!)

b) $J = \frac{i}{a} = \frac{i}{\pi r^2} = \frac{1530\,A}{\pi(3\times10^{-3}m)^2} = \boxed{5.41\times10^{7}\,\frac{A}{m^2}}$

c) $R = \frac{\rho L}{A} = \frac{\rho L}{\pi r^2}$ so $\rho = \frac{\pi r^2 R}{L}$

$$\rho = \frac{\pi(3\times10^{-3}m)^2(15\times10^{-3}\Omega)}{4m} = 1.06\times10^{-7}\,\Omega\cdot m$$

From Table 27.1, the wire is made of platinum

(27-4) a) $J = \sigma E$ and $J = (n_+ + n_-)e v_d$

(Note that the motion of both the positive and negative ions count as positive current flow, since they are in opposite directions)

$$\sigma E = (n_+ + n_-)e v_d$$

$$v_d = \frac{\sigma E}{(n_+ + n_-)e}$$

$$= \frac{(2.7\times10^{-14}\,1/\Omega m)(120\,V/m)}{(620\,1/cm^3 + 550\,1/cm^3)\left(\frac{100\,cm}{1m}\right)^3(1.6\times10^{-19}C)}$$

$$\boxed{v_d = 1.73 \times 10^{-2}\ \text{m/sec}}$$

b) $J = \sigma E = (2.7 \times 10^{-14} \frac{1}{\Omega \cdot m})(120 \frac{V}{m}) = \boxed{3.24 \times 10^{-12} \frac{A}{m^2}}$

(27-5) Using $P = iV$,

$$\Delta q = i \Delta t = \frac{P}{V} \Delta t = \left(\frac{7\,W}{9\,V}\right)(5\,hr)\left(\frac{3600\,sec}{1\,hr}\right)$$

$$= \boxed{1.40 \times 10^{4}\ C}$$

(27-6) a) $\#\ \text{electrons} = \frac{\Delta q}{e} = \frac{i \Delta t}{e} = \frac{(0.5A)(0.1 \times 10^{-6} sec)}{1.6 \times 10^{-19} C/\text{electron}}$

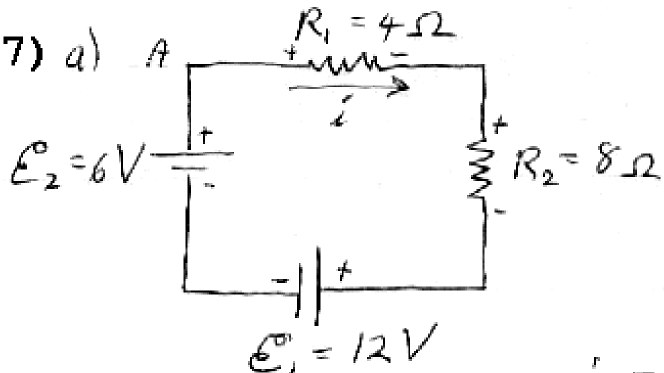
$$= \boxed{3.13 \times 10^{11}\ \text{electrons}}$$

b) ave $i_{ave} = \frac{\Delta q}{\Delta t} = \frac{(500)(0.5A)(0.1 \times 10^{-6} sec)}{1\,sec} = \boxed{2.5 \times 10^{-5}\ A}$

c) peak power $P = iV = (0.5\,A)(50 \times 10^{6}\,V) = \boxed{2.5 \times 10^{7}\,W}$

ave power $P_{ave} = i_{ave} V = (2.5 \times 10^{-5} A)(50 \times 10^{6} V) = \boxed{1250\,W}$

(28-7) a)



Going clockwise from A,

$$-iR_1 - iR_2 - \mathcal{E}_1 + \mathcal{E}_2 = 0$$

$$-i(R_1 + R_2) = \mathcal{E}_1 - \mathcal{E}_2$$

$$i = \frac{-(\mathcal{E}_1 - \mathcal{E}_2)}{R_1 + R_2} = \frac{-(12V - 6V)}{4\Omega + 8\Omega} = \boxed{-0.5A}$$

The current goes the other way counterclockwise

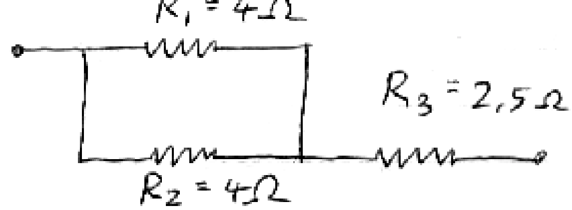
b) $P_1 = i^2 R_1 = (0.5\,A)^2 (4\Omega) = \boxed{1.0\,W}$

$P_2 = i^2 R_2 = (0.5A)^2 (8\Omega) = \boxed{2.0\,W}$

c) Battery 1 is discharging: $P = iV = (0.5\,A)(12\,V) = \boxed{6.0\,W}$

Battery 2 is charging: $P = iV = (0.5\,A)(6\,V) = \boxed{3.0\,W}$

(28-8)



R_1 and R_2 are in parallel, so

$$R_{12} = \frac{R_1 R_2}{R_1 + R_2} = \frac{(4\Omega)(4\Omega)}{4\Omega + 4\Omega}$$

$$R_{12} = 2\,\Omega$$

R_{12} and R_3 are in series, so $R_{123} = R_{12} + R_3$

$$= 2\,\Omega + 2.5\,\Omega$$

$$= \boxed{4.5\,\Omega}$$

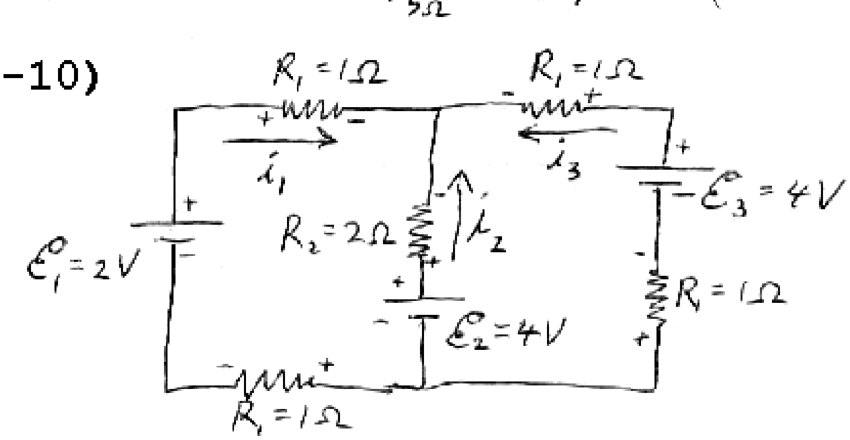
(28-9) There are $\mathcal{E} = 12\,V$ across the 3Ω and 5Ω resistors, which are in series. The current through these resistors is therefore

$$i = \frac{\mathcal{E}}{3\Omega + 5\Omega} = \frac{12\,V}{3\Omega + 5\Omega} = 1.5\,A$$

The voltage across the 5Ω resistor is therefore

$$V_{5\Omega} = iR = (1.5\,A)(5\,\Omega) = \boxed{7.5\,V}$$

(28-10)



At the top center junction,

$$\boxed{\underbrace{i_1 + i_2 + i_3}_{\text{current in}} = 0}$$

current in = current out

For the left loop, going around clockwise,

$$\boxed{-i_1 R_1 + i_2 R_2 - \mathcal{E}_2 - i_1 R_1 + \mathcal{E}_1 = 0}$$

For the right loop, going around clockwise,

$$\boxed{i_3 R_1 - \mathcal{E}_3 + i_3 R_1 + \mathcal{E}_2 - i_2 R_2 = 0}$$

Plugging in values for the resistors, the boxed equations above are

$$-i_1(2\Omega) + i_2(2\Omega) = -\mathcal{E}_1 + \mathcal{E}_2 = -2V + 4V = 2V$$

$$\boxed{-i_1(2\Omega) + i_2(2\Omega) = 2V} \ *$$

$$i_3(2\Omega) - i_2(2\Omega) = \mathcal{E}_3 - \mathcal{E}_2 = 4V - 4V = 0$$

$$\boxed{i_3(2\Omega) - i_2(2\Omega) = 0} \ **$$

And we still have $\boxed{i_1 + i_2 + i_3 = 0}$ ***

Solve equation *** for i_3, and substitute into **

$$i_3 = -(i_1 + i_2) \quad \text{so} \quad -(i_1 + i_2)(2\Omega) - i_2(2\Omega) = 0$$

$$\Rightarrow \quad -(i_1 + i_2) - i_2 = 0 \qquad \text{(cancel the } 2\Omega\text{)}$$

$$i_1 = -2i_2$$

Now plug this into * :

$$-(-2i_2)(2\Omega) + i_2(2\Omega) = 2V$$

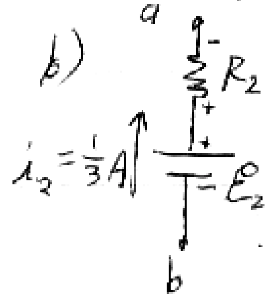
$$i_2(6\Omega) = 2V \Rightarrow \boxed{i_2 = \frac{1}{3}A}$$

Then $i_1 = -2i_2 = -2(\frac{1}{3}A) = \boxed{-\frac{2}{3}A}$

(i_1 is really going counterclockwise)

Finally, $i_3 = -(i_1 + i_2) = -(-\frac{2}{3}A + \frac{1}{3}A) = \boxed{\frac{1}{3}A}$

b)



(start at b)

$V_a - V_b = +\mathcal{E}_2 - i_2 R_2$

$= 4V - (\frac{1}{3}A)(2\Omega) = \boxed{3\frac{1}{3}V}$

(28-11) a) $\tau = RC = (1.4\times10^{6}\Omega)(1.8\times10^{-6}F) = \boxed{2.52\text{ sec}}$

b) $q_0 = \mathcal{E}C = (12V)(1.8\times10^{-6}F) = \boxed{2.16\times10^{-5}C}$

c) $q = q_0(1-e^{-t/RC}) \Rightarrow 1 - \frac{q}{q_0} = e^{-t/RC}$

So $\ln\left(\frac{q_0 - q}{q_0}\right) = -\frac{t}{RC}$

Using $-\ln x = \ln\frac{1}{x}$, this is

$$\frac{t}{RC} = \ln\left(\frac{q_0}{q_0 - q}\right)$$

$$t = \underset{\tau}{RC}\ln\left(\frac{q_0}{q_0-q}\right) = (2.52\text{ sec})\ln\left(\frac{21.6\mu C}{21.6\mu C - 16\mu C}\right)$$

$$= \boxed{3.40\text{ sec}}$$

(28-12) a) $q = q_0 e^{-t/\tau}$ and $V = \frac{q}{C}$, so dividing by C

$$\Rightarrow V = V_0 e^{-t/\tau}$$

$$\frac{V}{V_0} = e^{-t/\tau}$$

$$\ln\left(\frac{V}{V_0}\right) = -\frac{t}{\tau}$$

$$\tau = \frac{t}{\ln(V_0/V)} = \frac{10\text{ sec}}{\ln(100V/1V)} = \boxed{2.17\text{ sec}}$$

b) $V = V_0\, e^{-t/\tau}$

$$= 100V\, e^{-17\text{ sec}/2.17\text{ sec}}$$

$$= \boxed{3.96 \times 10^{-2}\ V}$$