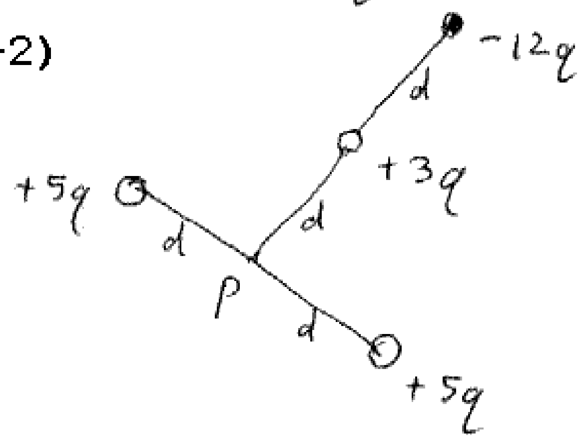
Homework #2

(23-1) $E = k\frac{|q|}{r^2}$ so $|q| = \frac{Er^2}{k}$

Thus $q = \frac{(1\ N/C)(1\ m)^2}{8.99\times10^9\ Nm^2/C^2} = \boxed{1.11\times10^{-10}\ C}$

(23-2)



By symmetry, the electric fields due to the two +5q charges cancels at point P (but not anywhere else).

That leaves the electric field at point P due to the +3q and -12q charges.

The electric field at P due to the +3q charge has magnitude

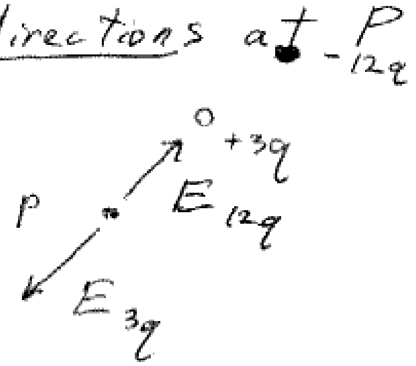
$$E_{3q} = k\frac{|3q|}{d^2} = 3k\frac{q}{d^2}$$

and the electric field at P due to the -12q charge has magnitude

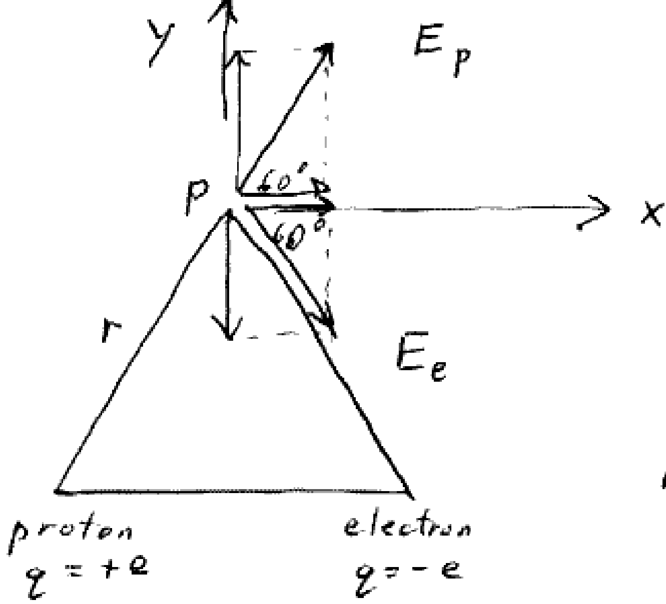
$$E_{12q} = k\frac{|-12q|}{(2d)^2} = 3k\frac{q}{d^2}$$

But these two electric fields point in opposite directions at P $\Rightarrow$ they cancel

$\Rightarrow$ total $\boxed{\vec{E} = 0}$



(23-3)



The magnitudes of E_p and E_e are the same at point P:

$$E_p = E_e = k\frac{e}{r^2}$$

The y-components should cancel, by symmetry. Let's check this:

$$\left.\begin{aligned} E_{p,y} &= k\frac{e}{r^2}\sin 60^\circ \\ E_{e,y} &= -k\frac{e}{r^2}\sin 60^\circ \end{aligned}\right\} \text{add to zero}$$

The two x-components add:

$$E_{p,x} = k\frac{e}{r}\cos 60^\circ$$

$$E_{e,x} = k\frac{e}{r^2}\cos 60^\circ$$

$$\Rightarrow E_{total} = E_{p,x} + E_{e,x} = 2k\frac{e}{r^2}\cos 60^\circ$$

$$= 2\left(8.99\times10^{9}\,\frac{Nm^2}{C^2}\right)\frac{1.6\times10^{-19}C}{(2\times10^{-6}m)^2}\cos 60^\circ$$

$$= \boxed{360\,\frac{N}{C}}$$

(23-4) The magnitude of the dipole moment vector is

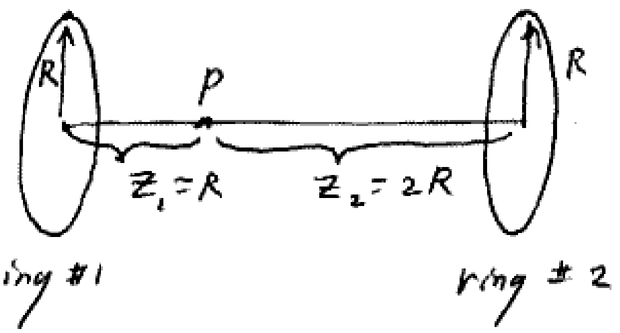
$$p = qd = (1.6\times10^{-19}C)(4.3\times10^{-9}m)$$

$$= 6.88\times10^{-28}\,C\cdot m$$

The dipole moment vector points from the electron toward the proton.

(23-5) The electric field due to a charged ring is of magnitude

$$E = k\, \frac{qz}{(z^2+R^2)^{3/2}}$$



Assuming that q_1 and q_2 are positive, E_1 points away from ring #1, and E_2 points away from ring #2. If the fields cancel at point P, then

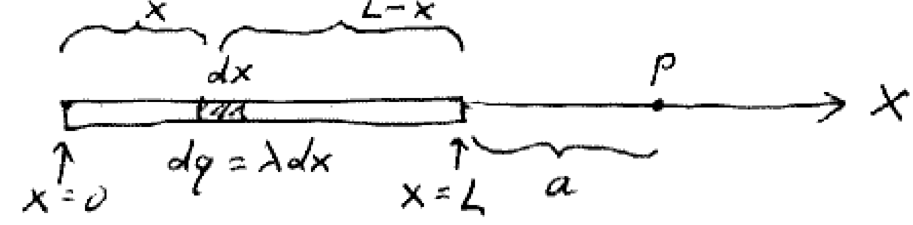
$$E_1 = E_2$$

$$k\, \frac{q_1 R}{(R^2+R^2)^{3/2}} = k\, \frac{2 q_2 R}{[(2R)^2+R^2]^{3/2}}$$

$$\frac{q_1}{q_2} = \frac{2(2R^2)^{3/2}}{(5R^2)^{3/2}} = 2\left(\frac{2}{5}\right)^{3/2} = \boxed{0.506}$$

(23-6) a) $\boxed{\lambda = -q/L}$

b) Put the left end of the rod at the origin.



Choose a little bit of length dx of the rod, which has charge $dq = \lambda\, dx$

Because dq looks like a point charge, it creates a little bit of electric field at point P of magnitude

$$dE = k\frac{dq}{(L-x+a)^2} = k\frac{\lambda\,dx}{(L-x+a)^2}$$

Each little bit of length dx creates a little bit of electric field in the $+x$ direction. To find the total field, we add up (integrate) all the dE's

$$\vec{E} = \int dE\ (\hat{i})$$

$$= \int_{x=0}^{x=L} k\frac{\lambda\,dx}{(L-x+a)^2}\ (\hat{i})$$

Integrate this: let $u = L-x+a$ so $du = -dx$

$$\vec{E} = -k\lambda\int\frac{du}{u^2}\ (\hat{i})$$

$$= -k\lambda\left(-\frac{1}{u}\right)\Big|_{x=0}^{x=L}\ (\hat{i})$$

$$= k\lambda\frac{1}{L-x+a}\Big|_{x=0}^{x=L}\ (\hat{i})$$

$$= k\lambda\left[\frac{1}{a} - \frac{1}{L+a}\right]\ (\hat{i})$$

$$= k\lambda\left[\frac{L+a}{a(L+a)} - \frac{a}{a(L+a)}\right]\ (\hat{i})$$

$$= k\frac{\lambda L}{a(L+a)}\ (\hat{i})$$

$$= k\frac{(-Q/L)L}{a(L+a)}\ (\hat{i})$$

$$\boxed{\vec{E} = -k\frac{Q}{a(L+a)}\hat{i}}$$

c) If $a >> L$, then $L + a \simeq a$ and

$$\boxed{\vec{E} \simeq -k \frac{q}{a^2} \hat{i}}$$

which looks like a point charge.

(23-7) a) $\vec{a} = \frac{\vec{F}}{m_e} = \frac{q_e \vec{E}}{m_e} = \frac{-e\vec{E}}{m_e}$

So $\vec{a} = \frac{-(1.6 \times 10^{-19} C)(120\ N/C)\hat{j}}{9.11 \times 10^{-31} Kg} = \boxed{-2.11 \times 10^{13} \frac{m}{sec^2} \hat{j}}$

b) Since $a_x = 0$, the time for the electron's x-coordinate to change by 2.0 cm comes from

$$x - x_o = V_{0x} t + \frac{1}{2} \cancelto{0}{a_x} t^2 = V_{0x} t$$

$$\Rightarrow t = \frac{x - x_o}{V_{0x}} = \frac{0.02\ m}{1.5 \times 10^5\ m/sec} = 1.33 \times 10^{-7} sec$$

The x-component of velocity doesn't change (since $a_x = 0$), but the y-component of velocity changes to

$$V_y = V_{0y} + a_y t$$

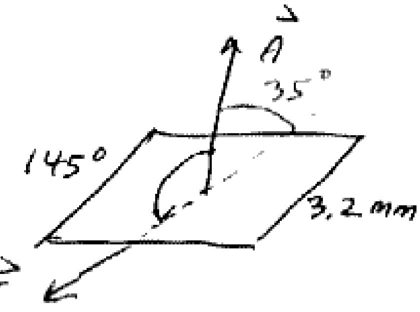
$$= 3 \times 10^3 \frac{m}{sec} + (-2.11 \times 10^{13} \frac{m}{sec^2})(1.33 \times 10^{-7} sec)$$

$$= -2.81 \times 10^6 \frac{m}{sec}$$

So the electron's velocity vector is

$$\boxed{\vec{V} = 1.9 \times 10^5 \frac{m}{sec} \hat{i} - 2.81 \times 10^6 \frac{m}{sec} \hat{j}}$$

(24-8) $\Phi = \int \vec{E} \cdot d\vec{A} = \int E\, dA \cos\theta$



Since E and θ are constant,

$$\Phi = EA\cos\theta$$

$$= \left(1800\ \tfrac{N}{C}\right)\left(3.2\times10^{-3}\,m\right)^2 \cos 145°$$

$$= \boxed{-1.51\times10^{-2}\ \frac{Nm^2}{C}}$$

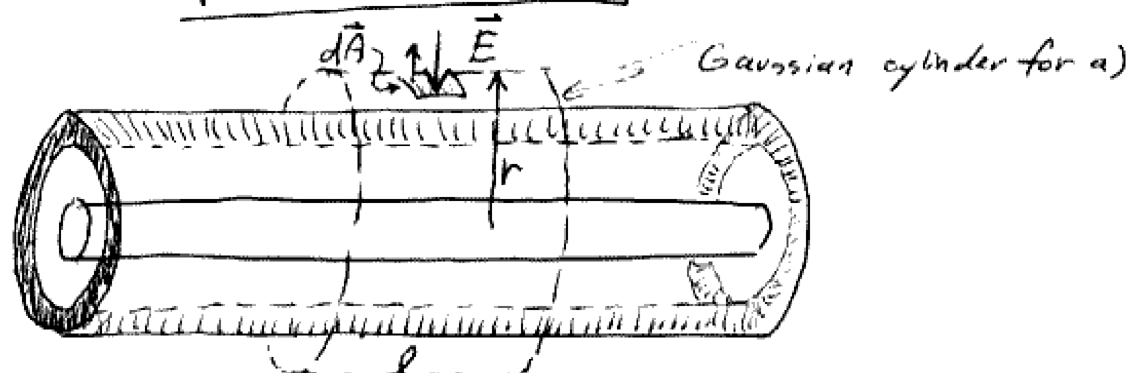
(24-9) Above a conducting surface,

$$E = \frac{\sigma}{\varepsilon_0} \quad \text{or} \quad \sigma = \varepsilon_0 E$$

So $\sigma = \left(8.85\times10^{-12}\ \frac{C^2}{Nm^2}\right)\left(2.3\times10^{5}\ \frac{N}{C}\right)$

$$= \boxed{2.04\times10^{-6}\ \frac{C}{m^2}}$$

(24-10)



The linear charge density of the central rod is

$$\lambda_{rod} = \frac{+q}{L}$$

and the linear charge density of the cylindrical shell is

$$\lambda_{shell} = \frac{-2q}{L}$$

a) Draw a Gaussian cylinder outside the conducting shell, of length l. The charge enclosed by the Gaussian cylinder is

$$q_{enc} = (\lambda_{rod} + \lambda_{shell})\, l$$
$$= \left(\frac{+q}{L} - \frac{2q}{L}\right) l = -\frac{ql}{L}$$

Start with Gauss' Law,

$$\varepsilon_0 \oint E\, dA \cos\theta = q_{enc}$$

The electric field points straight <u>toward</u> the cylinders (since q_{enc} is negative), so $\theta = 180°$ (between dA and E) on the side of the Gaussian cylinder. There is no flux through the ends (since E is parallel to the ends of the Gaussian cylinder). We only have to consider the side of the cylinder, so

$$\varepsilon_0 \int_{\substack{\text{side of} \\ \text{Gaussian cylinder}}} E\, dA\, \underbrace{\cos 180°}_{-1} = -\frac{ql}{L}$$

By symmetry, E has the same magnitude everywhere over the side of the Gaussian cylinder, so we can pull it outside the integral

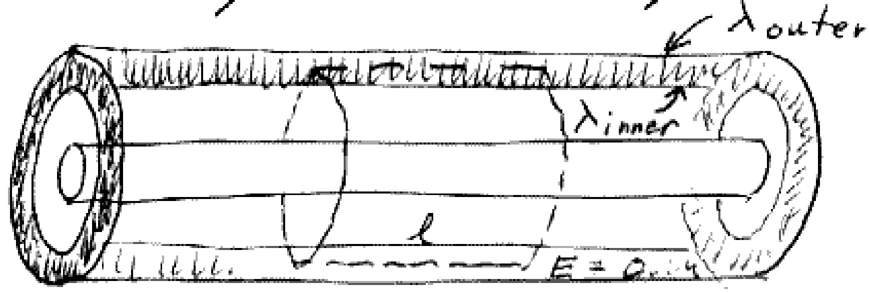
$$\varepsilon_0 E \int_{\substack{\text{side of} \\ \text{Gaussian cylinder}}} dA = \frac{ql}{L}$$

But $\int dA$ = area of Gaussian cylinder side
$= 2\pi r l$

Thus $\varepsilon_0 E \, 2\pi r l = \frac{q l}{L}$

$$\boxed{E = \frac{q/L}{2\pi \varepsilon_0 r}} \quad \left(\text{or } \frac{q}{2\pi \varepsilon_0 r L}\right) \underline{\text{inward}}$$

b) Draw a Gaussian cylinder within the conducting shell, of length l.



Let λ_{inner} = linear charge density on the inner surface of the conducting shell

λ_{outer} = linear charge density on the outer surface of the conducting shell

so $\lambda_{inner} + \lambda_{outer} = \lambda_{shell} = -\frac{2q}{L}$

There is no flux through the ends of the Gaussian cylinder, and the sides of the cylinder are within the conducting shell, where $\vec{E} = 0$.

$\Rightarrow$ there is no flux through the Gaussian cylinder

$\Rightarrow \Phi = 0$ so $\frac{q_{enc}}{\varepsilon_0} = 0$

But $q_{enc} = (\lambda_{rod} + \lambda_{inner}) l$

$$\Rightarrow (\lambda_{rod} + \lambda_{inner})\ell = 0$$

$$\Rightarrow \boxed{\lambda_{inner} = -\lambda_{rod} = -\frac{q}{L}}$$

Then $\lambda_{inner} + \lambda_{outer} = -\frac{2q}{L}$ gives

$$\lambda_{outer} = -\frac{2q}{L} - \lambda_{inner}$$

$$= -\frac{2q}{L} - \left(-\frac{q}{L}\right)$$

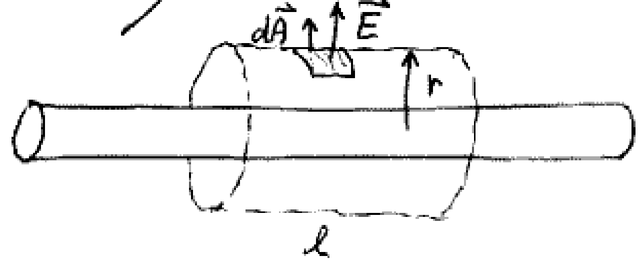
$$\boxed{\lambda_{outer} = -\frac{q}{L}}$$

The actual charges on the surfaces of the cylindrical shell are

$$q_{inner} = \lambda_{inner} L = -q$$

$$q_{outer} = \lambda_{outer} L = -q$$

c) Draw a Gaussian cylinder between the conducting shell and the rod. The conducting shell lies entire outside the Gaussian cylinder, so we can ignore it!



This part goes just like part a), but ignore the conducting shell.

$$\varepsilon_0 \oint E\, dA \cos\theta = q_{enc}$$

with $q_{enc} = \lambda_{rod}\, l = \frac{+ql}{L}$

and $\theta = 0°$ since $\vec{E}$ points away from the positive q_{enc}

So, since there is no flux through the ends of the cylinder,

$$\varepsilon_0 \int_{\substack{\text{side of} \\ \text{Gaussian cylinder}}} E\, dA \underbrace{\cos 0°}_{=1} = \frac{ql}{L}$$

By symmetry, E has the same magnitude everywhere over the side of the Gaussian cylinder, so we can pull it outside the integral:

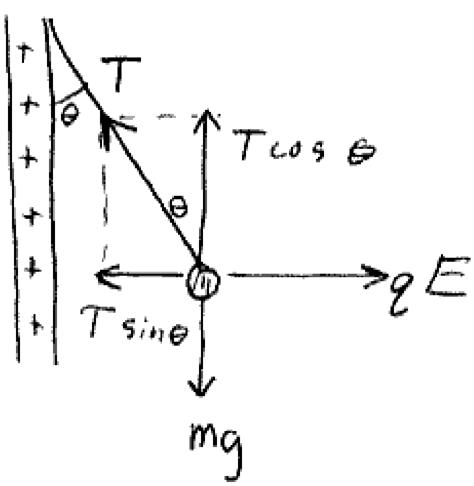
$$\varepsilon_0 E \int dA = \frac{ql}{L}$$

But $\int dA = 2\pi r l$ as before, so

$$\varepsilon_0 E\, 2\pi r l = \frac{ql}{L}$$

$$\boxed{E = \frac{q/L}{2\pi \varepsilon_0 r}}$$ <u>outward</u>

(24-11)



The electric field due to a sheet of charge is

$$E = \frac{\sigma}{2\varepsilon_0}$$

The electric force, tension in the thread, and gravitational force are all in balance.

$$T\sin\theta = qE \quad \text{and} \quad T\cos\theta = mg$$

Divide these: $\dfrac{T\sin\theta}{T\cos\theta} = \dfrac{qE}{mg}$

$$\tan\theta = \frac{qE}{mg} = \frac{q}{mg}\frac{\sigma}{2\varepsilon_0}$$

So $\quad \sigma = \dfrac{2mg\varepsilon_0}{q}\tan\theta$

$$= \frac{2(10^{-6}\,kg)(9.8\,\frac{m}{sec^2})(8.85\times10^{-12}\,C^2/Nm^2)}{2\times10^{-8}\,C}$$

$$\times \tan 30^\circ$$

$$\boxed{\sigma = 5.01\times10^{-9}\,\frac{C}{m^2}}$$

(24-12) a) Draw the Gaussian sphere of radius r inside the central uniformly charged sphere, We can ignore everything else outside!

$$\varepsilon_0 \oint E\,dA\cos\theta = q_{enc}$$

Here, $\theta = 0$ since $\vec{E}$ and $d\vec{A}$ are both radially outward. What is q_{enc}?

$q_{enc} = \left(\begin{matrix}\text{charge per}\\ \text{unit volume}\end{matrix}\right) \times$ volume of Gaussian sphere

$$= \frac{q}{\frac{4}{3}\pi a^3}\left(\frac{4}{3}\pi r^3\right)$$

$$= q\left(\frac{r^3}{a^3}\right)$$

So $\varepsilon_0 \oint E\,dA \underbrace{\cos 0°}_{1} = q\left(\frac{r^3}{a^3}\right)$

Since E is constant over the Gaussian sphere, we pull it outside the integral

$$\varepsilon_0 E \oint dA = q\left(\frac{r^3}{a^3}\right)$$

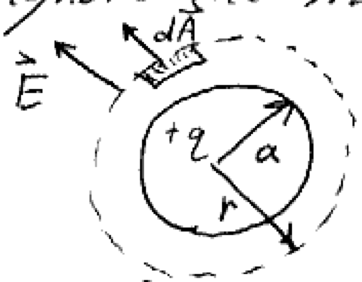
But $\oint dA$ = surface area of Gaussian sphere
$= 4\pi r^2$

so $\varepsilon_0 E\, 4\pi r^2 = q\left(\frac{r^3}{a^3}\right)$

$$\boxed{E = \frac{1}{4\pi\varepsilon_0} \frac{q r}{a^3}}$$ outward

for $r < a$.

b) Draw the Gaussian sphere of radius r between the central sphere and the shell. This means we can ignore the shell.



$$\varepsilon_0 \oint E\,dA \cos\theta = q_{enc} = +q$$

Again, E is constant over the Gaussian sphere and $\theta = 0°$, so

$$\varepsilon_0 E \oint dA = +q$$

As before, $\oint dA = 4\pi r^2$, so

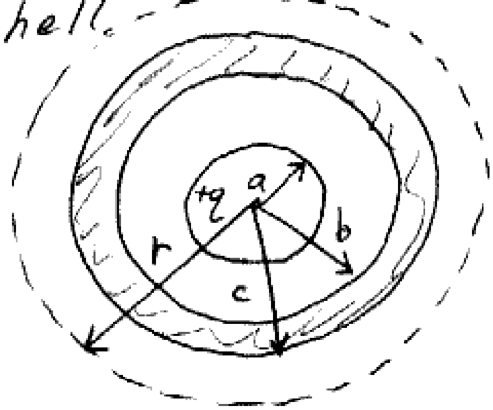
$$\varepsilon_0 E\, 4\pi r^2 = q$$

$$\boxed{E = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}} \text{ outward}$$

for $a < r < b$. It look like a point charge at the center.

c) Inside the conducting shell, $\boxed{\vec{E} = 0}$

d) Draw the Gaussian sphere of radius r outside the shell.

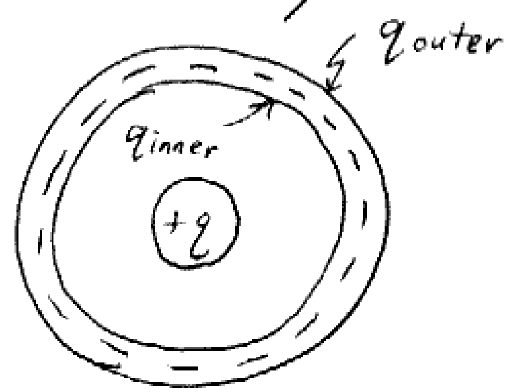


$-q$ on conducting shell

$$\varepsilon_0 \oint E\, dA \cos\theta = q_{enc} = q + (-q) = 0$$

$$\Rightarrow \boxed{\vec{E} = 0} \text{ for } r > c$$

e) Draw the Gaussian sphere inside the conducting shell, where $\vec{E} = 0$



$q_{outer} + q_{inner} = -q$

Here, since $\vec{E} = 0$,

$\Phi = 0$ so $\frac{q_{enc}}{\varepsilon_0} = 0$

But $q_{enc} = q + q_{inner}$

so $q + q_{inner} = 0$

or $\boxed{q_{inner} = -q}$ on inner surface of conducting shell

Now use $q_{outer} + q_{inner} = -q$

$q_{outer} - q = -q$

$\boxed{q_{outer} = 0}$ no charge is on the outer surface of the conducting shell