Homework #13

(41-1) There are $2n^2$ electron states in shell n

a) $2n^2 = 2(4)^2 = 32$

b) $2n^2 = 2(1)^2 = 2$

c) $2n^2 = 2(3)^2 = 18$

d) $2n^2 = 2(2)^2 = 8$

(41-2) $n = 1, \ \ell = 0, \ m_\ell = 0, \ m_s = +\frac{1}{2}$

$n = 1, \ \ell = 0, \ m_\ell = 0, \ m_s = -\frac{1}{2}$

(43-3) initial:

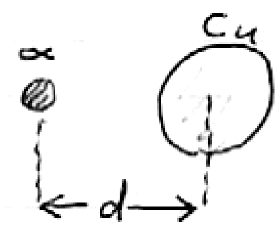
$K_\alpha = 5.30 \text{ MeV}$ $\qquad$ $K_{Cu} = 0$

$U = 0$

final: $K_\alpha = 0$

$K_{Cu} = 0$

$U = \frac{1}{4\pi\varepsilon_0} \frac{q_\alpha q_{Cu}}{d}$



$q_\alpha = 2e$ $\qquad$ $q_{Cu} = 29e$

By conservation of energy

$$5.30 \text{ MeV} = \frac{1}{4\pi\varepsilon_0} \frac{q_\alpha q_{Cu}}{d}$$

$$d = \frac{1}{4\pi\varepsilon_0} \frac{q_\alpha q_{Cu}}{5.30 \text{ MeV}}$$

Recalling that $\frac{1}{4\pi\varepsilon_0} = 9 \times 10^9 \frac{Nm^2}{C^2}$, we find

$$d = \left(9\times10^{9}\,\frac{Nm^2}{C^2}\right)\frac{(2)(29)(1.6\times10^{-19}C)^2}{(5.30\times10^{6}eV)\left(\frac{1.6\times10^{-19}J}{1eV}\right)}$$

$$= 1.58\times10^{-14}m = \boxed{15.8\ fm}$$

(43-4) $\Delta x\,\Delta p \simeq \hbar$ and $\Delta p = m\Delta V$ (m = proton or neutron mass)

So the uncertainty in the nucleon's velocity is

$$\Delta V \simeq \frac{\hbar}{m\Delta x} = \frac{h/2\pi}{m\,\Delta x}$$

Using $\Delta x \simeq R = R_o A^{1/3} = 1.2\,fm\,(100)^{1/3} = 5.57\,fm$,
this is

$$\Delta V \simeq \frac{(6.63\times10^{-34}\,J\cdot sec)/2\pi}{(1.67\times10^{-27}kg)(5.57\times10^{-15}m)}$$

$$= 1.13\times10^{7}\,\frac{m}{sec}$$

The value of the nucleon's speed must be at least $1.13\times10^{7}\frac{m}{sec}$. Otherwise, since the uncertainty in the speed is $\pm 1.13\times10^{7}\frac{m}{sec}$, we could have a negative value for a measurement of the speed. Setting $V = 1.13\times10^{7}\frac{m}{sec}$, the kinetic energy is

$$K \simeq \tfrac{1}{2}mV^2 = \tfrac{1}{2}(1.67\times10^{-27}kg)(1.13\times10^{7}\tfrac{m}{sec})^2$$

or

$$K = 1.07\times10^{-13}J\left(\frac{1eV}{1.6\times10^{-19}J}\right)$$

$$= \boxed{672\ keV}$$

(43-5) The mass of a gallium atom, ^{67}Ga, is

$$m = 67u = 67(1.66\times10^{-27}\,Kg) = 1.11\times10^{-25}\,Kg$$

In this 3.4 g sample, the initial number of ^{67}Ga atoms is

$$N_o = \frac{3.4\times10^{-3}\,Kg}{1.11\times10^{-25}\,Kg/atom} = 3.06\times10^{22}\text{ atoms}$$

The disintegration constant λ is

$$\lambda = \frac{\ln 2}{\tau} = \frac{\ln 2}{(78\,hr)\left(\frac{3600\,sec}{1\,hr}\right)} = 2.47\times10^{-6}\,\frac{1}{sec}$$

a) The initial decay rate is

$$R_o = \lambda N_o = \left(2.47\times10^{-6}\,\frac{1}{sec}\right)\left(3.06\times10^{22}\text{ atoms}\right)$$

$$\boxed{R_o = 7.56\times10^{16}\,\frac{decays}{sec}}$$

b) $R = R_o e^{-\lambda t} = R_o e^{-\ln(2)t/\tau}$

So $R = 7.56\times10^{16}\,\frac{decays}{sec}\, e^{-\ln(2)(48\,hr)/(78\,hr)}$

$$\boxed{R = 4.93\times10^{16}\,\frac{decays}{sec}}$$

(43-6) a) $\tau = \frac{\ln 2}{\lambda} = \frac{\ln 2}{0.0108\,hr^{-1}} = \boxed{64.2\,hr}$

b) After 3 half-lives, the number of ^{197}Hg atoms remaining is

$$N = \left(\tfrac{1}{2}\right)^3 N_o = \tfrac{1}{8} N_o$$

One-eighth ~~8th~~ of the sample remains.

c) $\frac{N}{N_0} = e^{-\lambda t} = e^{-(0.0108\,hr^{-1})(10\,d)(\frac{24\,hr}{1\,d})}$

$= \boxed{0.0749}$

(43-7) $Q = [(\text{mass in}) - (\text{mass out})]c^2$

$= [m(^{238}U) - m(^{234}Th) - m(^4He)]c^2$

$= [238.05079\,u - 234.04363\,u - 4.00260\,u]c^2$

$= 4.56 \times 10^{-3}\,uc^2$

Using $uc^2 = 931.5$ MeV, we find

$Q = (4.56 \times 10^{-3})(931.5\,MeV)$

$= \boxed{4.25\,MeV}$

(44-8) a) The mass of the ^{235}U atom is

$m = 235.04392\,u$

$= 235.04392\,(1.661 \times 10^{-27}\,kg)$

$= 3.904 \times 10^{-25}\,kg$

The number of ^{235}U atoms in 1 kg is

$N = \frac{1\,kg}{3.904 \times 10^{-25}\,kg/atom} = \boxed{2.56 \times 10^{24}\,atoms}$

b) The energy released in the fissioning of 1 kg of ^{235}U is

$$E = NQ = (2.56\times10^{24})(2\times10^{8}\,eV)\left(\frac{1.6\times10^{-19}\,J}{1\,eV}\right)$$

$$= \boxed{8.19\times10^{13}\,J}$$

c) $$t = \frac{E}{power} = \frac{8.19\times10^{13}\,J}{100\,J/sec} = 8.19\times10^{11}\,sec$$

or $$t = (8.19\times10^{11}\,sec)\left(\frac{1\,yr}{3.16\times10^{7}\,sec}\right)$$

$$= \boxed{2.59\times10^{4}\,years}$$

(44-9) $$Q = [(mass\ in) - (mass\ out)]c^2$$

$$= [m(^{235}U) + m_n - m(^{141}Cs) - m(^{93}Rb) - 2m_n]c^2$$

$$= [235.04392\,u - 1.00867\,u - 140.91963\,u - 92.92157\,u]c^2$$

$$= 0.194\,uc^2$$

$$= (0.194)(931.5\,MeV) = \boxed{181\,MeV}$$

(44-10) This is like (44-8). The number of ^2H atoms in 1 kg is

$$N = \frac{1\,kg}{mass\ of\ ^2H\ atom}$$

$$= \frac{1\,kg}{(2.014102\,u)(1.661\times10^{-27}\,kg/u)} = 2.99\times10^{26}\,atoms$$

A fusion event involves a pair of ^2H ($^2H + {}^2H \rightarrow {}^3He + n$), so the number of pairs of ^2H atoms is $N/2$.

Each fusion event releases $Q = 3.27\,MeV$, so the energy released is

$$E = \left(\frac{N}{2}\right)(Q)$$

$$= \left(\frac{2.99\times10^{26}}{2}\right)(3.27\times10^{6}\,eV)\left(\frac{1.6\times10^{-19}\,J}{1\,eV}\right)$$

$$= 7.82\times10^{13}\,J$$

The 100 watt lamp would burn for a time

$$t = \frac{7.82\times10^{13}\,J}{100\,J/sec}\left(\frac{1\,yr}{3.16\times10^{7}\,sec}\right) = \boxed{2.48\times10^{4}\text{ years}}$$

This is about 3×10^{4} years.

(44-11) $Q = [(\text{mass in}) - (\text{mass out})]c^2$

$$= [3m(^4He) - m(^{12}C)]c^2$$

$$= [3(4.0026\,u) - 12.0000\,u]c^2$$

$$= 7.8\times10^{-3}\,uc^2 = 7.8\times10^{-3}(931.5\,MeV) = \boxed{7.27\,MeV}$$

(44-12) a) $$\frac{\text{mass}}{\text{sec}} = \frac{\text{energy/sec}}{c^2} = \frac{3.9\times10^{26}\,J/sec}{(3\times10^{8}\,m/sec)^2} = \boxed{4.33\times10^{9}\,\frac{Kg}{sec}}$$

b) In 4.5×10^{9} yrs, the Sun has lost a mass

$$m = (4.5\times10^{9}\,yrs)\left(\frac{3.16\times10^{7}\,sec}{1\,yr}\right)\left(4.33\times10^{9}\,\frac{Kg}{sec}\right)$$

$$= \boxed{6.16\times10^{26}\,Kg}$$

This is only $\frac{6.16\times10^{26}\,Kg}{2.0\times10^{30}\,Kg} = \boxed{3.08\times10^{-4}}$ of the Sun's mass