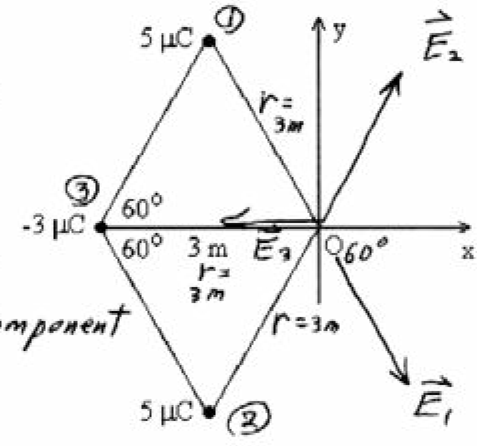
Name: KEY

PHYSICS 2220 - SPRING 2009 - FINAL EXAM

*** * * * WORK ANY FIVE OF THE FOLLOWING SEVEN PROBLEMS * * * ***

PROBLEMS TO BE GRADED (CIRCLE): 1 2 3 4 5 6 7

1. Two 5 μC charges and a -3 μC charge are arranged as shown. Each side of the two equilateral triangles is 3 m long.



   a. Find the electric field vector and the electric potential at the origin, O.

By symmetry, $\vec{E}$ has no y-component

$$E_x = \frac{1}{4\pi\varepsilon_0}\frac{q_1}{r^2}\cos 60^\circ + \frac{1}{4\pi\varepsilon_0}\frac{q_2}{r^2}\cos 60^\circ - \frac{1}{4\pi\varepsilon_0}\frac{q_3}{r^2}$$

$$E_x = \frac{1}{4\pi\varepsilon_0}\frac{1}{r^2}\left[(q_1+q_2)\cos 60^\circ - q_3\right]$$

$$E_x = \left(9\times10^9\frac{Nm^2}{C^2}\right)\left(\frac{1}{(3m)^2}\right)\left[10\times10^{-6}C(\cos 60^\circ) - 3\times10^{-6}C\right)$$

$$\boxed{E_x = 2000\,\frac{N}{C} \text{ in } +x \text{ direction}}$$

$$V = \frac{1}{4\pi\varepsilon_0}\frac{q_1+q_2+q_3}{r} = \left(9\times10^9\frac{Nm^2}{C^2}\right)\frac{7\times10^{-6}C}{(3m)} = \boxed{2.1\times10^4 V}$$

   b. A proton is held at the origin, O, and then released. Find its speed when it is a great distance from the origin (where V = 0).

$$K_i + U_i = K_f + U_f \quad \text{with } K_i = 0$$

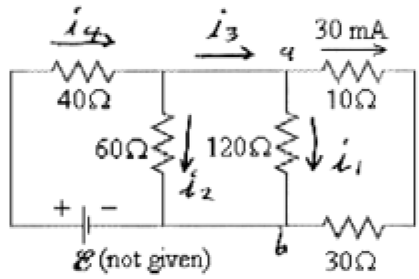
$$K_f = U_i - U_f = q(V_i - V_f) \quad \text{with } V_f = 0$$

$$\frac{1}{2}m_p v^2 = qV_i$$

$$\text{So } v = \sqrt{\frac{2qV_i}{m_p}} = \sqrt{\frac{2(1.6\times10^{-19}C)(2.1\times10^4V)}{1.67\times10^{-27}Kg}}$$

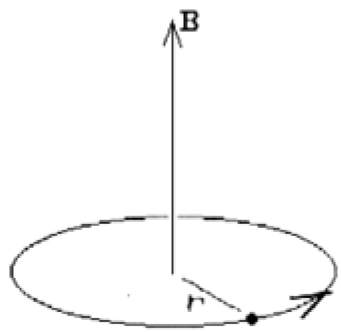
$$\boxed{v = 2\times10^6\,\frac{m}{sec}}$$

2. a. A current of 30 mA flows through the 10 Ω resistor. Find the current through the 40 Ω resistor.



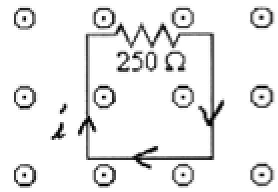
$$V_{ab} = 30\,mA(10\,\Omega + 30\,\Omega)$$
$$= 1200\,mV = 1.2\,V$$
$$i_1 = \frac{V_{ab}}{120\,\Omega} = \frac{1.2\,V}{120\,\Omega} = 1\times10^{-2}\,A = 10\,mA$$
$$i_2 = \frac{V_{ab}}{60\,\Omega} = \frac{1.2\,V}{60\,\Omega} = 2\times10^{-2}\,A = 20\,mA$$
$$i_3 = 30\,mA + i_1 = 30\,mA + 10\,mA = 40\,mA$$
$$i_4 = i_2 + i_3 = 20\,mA + 40\,mA = \boxed{60\,mA}$$

b. An electron travels in a circle perpendicular to a uniform magnetic field. The time for one orbit is 4.47×10^{-11} s. Find the strength of the magnetic field and <u>draw the direction of the electron's motion on the figure</u>.



$$r = \frac{mv}{qB} \text{ and } v = \frac{2\pi r}{T}$$
$$\text{So } r = \frac{m}{qB}\left(\frac{2\pi r}{T}\right)$$
$$B = \frac{2\pi m_e}{qT}$$
$$B = \frac{2\pi(9.11\times10^{-31}\,kg)}{(1.6\times10^{-19}\,C)(4.47\times10^{-11}\,sec)}$$
$$\boxed{B = 0.8\,T}$$

3. a. Each side of a square loop of wire (one turn) is 81 cm long. A 250 Ω resistor is in one side of the loop, and the loop is in a uniform magnetic field of 0.04 T directed out of the page. The magnetic field is smoothly increased to 0.11 T in 3.6 ms. Find the power dissipated by the resistor while the magnetic field is increased. <u>Draw the direction of the induced current on the diagram</u>.



$$\mathcal{E} = -N\frac{\Delta(BA\cos\theta)}{\Delta t} = -NA\cos 0^\circ \frac{B_f - B_i}{\Delta t}$$

$$\mathcal{E} = -1(0.81m)^2 \frac{0.11T - 0.04T}{3.6\times 10^{-3}sec} = -12.7575V$$

$$\Rightarrow i = \frac{\mathcal{E}}{R} = \frac{12.7575V}{250\Omega} = 5.103\times 10^{-2}A$$

$$\text{power } P = i^2 R = (5.103\times 10^{-2}A)^2(250\Omega)$$

$$\boxed{P = 0.651\ W}$$

b. An RLC series circuit has R = 20 Ω, L = 0.1 H, and C = 40 μF. The maximum emf of the alternating current power supply is $\mathcal{E}_m$ = 12 V. Find the maximum voltage across the resistor, the inductor, and the capacitor when the frequency of the power supply is set to 60 Hz.

$$\omega = 2\pi f = 2\pi\left(60\tfrac{1}{sec}\right) = 377\tfrac{rad}{sec}$$

$$Z = \sqrt{R^2 + \left(\omega L - \tfrac{1}{\omega C}\right)^2}$$

$$Z = \sqrt{(20\Omega)^2 + \left[(377\tfrac{rad}{sec})(0.1H) - \tfrac{1}{(377\frac{rad}{sec})(40\times10^{-6}F)}\right]^2}$$

$$Z = 34.91\Omega$$

$$\text{So } I_m = \frac{\mathcal{E}_m}{Z} = \frac{12V}{34.91\Omega} = 0.3437A$$

$$\Rightarrow V_R = I_m R = (0.3437A)(20\Omega) = \boxed{6.87\ V}$$

$$V_L = I_m \omega L = (0.3437A)(377\tfrac{rad}{sec})(0.1H) = \boxed{12.96\ V}$$

$$V_C = \frac{I_m}{\omega C} = \frac{0.3437A}{(377\frac{rad}{sec})(40\times10^{-6}F)} = \boxed{22.8\ V}$$

4. a. A lens made of glass (index of refraction 1.5) has a concave side whose radius of curvature is 40 cm, and a convex side whose radius of curvature is 15 cm. An object is placed 30 cm from the lens. Find the location of the image. Is the image <u>real</u> or <u>virtual</u>?

$$\frac{1}{f} = (n-1)\left(\frac{1}{R_1} + \frac{1}{R_2}\right) = (1.5-1)\left(-\frac{1}{40\,cm} + \frac{1}{15\,cm}\right)$$

$$\frac{1}{f} = \frac{1}{2}\left(-\frac{3}{120\,cm} + \frac{8}{120\,cm}\right) = \frac{5}{240\,cm}$$

$$\text{So } f = \frac{240\,cm}{5} = 48\,cm$$

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{i} \Rightarrow \frac{1}{i} = \frac{1}{f} - \frac{1}{p} = \frac{p-f}{pf}$$

$$\text{So } i = \frac{pf}{p-f} = \frac{(30\,cm)(48\,cm)}{30\,cm - 48\,cm} = \boxed{-80\,cm}$$

This is a $\boxed{\text{virtual image}}$

b. A diffraction grating has 125 lines per millimeter. Red light (λ = 700 nm) and blue light (λ = 400 nm) both pass through the grating and fall on a screen 4 m away. Find the distance on the screen between the red and blue 1st order maxima.

$$d = \frac{1}{125}\,mm = 8\times10^{-3}\,mm = 8\times10^{-6}\,m$$

$$d\sin\theta = d\left(\frac{y}{L}\right) = m\lambda \text{ with } m=1$$

$$\text{So } y_{red} = \frac{L\lambda_{red}}{d} = \frac{(4\,m)(700\times10^{-9}\,m)}{8\times10^{-6}\,m} = 0.35\,m$$

$$y_{blue} = \frac{L\lambda_{blue}}{d} = \frac{(4\,m)(400\times10^{-9}\,m)}{8\times10^{-6}\,m} = 0.20\,m$$

$$\text{Thus } y_{red} - y_{blue} = 0.35\,m - 0.20\,m = \boxed{0.15\,m}$$

5. a. Astronaut Amy travels to a distant star in her starship while couch-potato Brent stays on Earth. As the ship travels, Amy measures the ship's length as 120 m and Brent measures the ship's length as 96 m. If Brent ages 5 years during Amy's trip to the star, how much does Amy age?

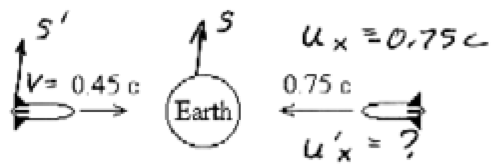
$$L_{moving} = \frac{L_{rest}}{\gamma} \quad \text{so} \quad \gamma = \frac{L_{rest}}{L_{moving}} = \frac{120\ m}{96\ m} = 1.25$$

$$\Delta t_{moving} = \Delta t_{rest}\, \gamma$$

$$\Delta t_{Brent} = \Delta t_{Amy}\, \gamma$$

$$\Rightarrow \Delta t_{Amy} = \frac{\Delta t_{Brent}}{\gamma} = \frac{5\ years}{1.25} = \boxed{4\ yrs}$$

b. Two spaceships approach Earth from opposite directions. An Earth observer determines that one ship is approaching with a speed of 0.75 c, and that the other spaceship is approaching with a speed of 0.45 c. How fast does each spaceship see the other spaceship moving?



$$u_x' = \frac{u - V}{1 - \frac{uV}{c^2}}$$

$$= \frac{-0.75c - 0.45c}{1 - \frac{(-0.75c)(0.45c)}{c^2}}$$

$$= \boxed{-0.897c}$$

the (−) sign means the S' ship sees the other ship moving in the −x' direction

6. a. Calculate the energies (in eV) and wavelengths of all possible photons that can be emitted when the electron cascades from the $n = 3$ to the $n = 1$ orbit of the hydrogen atom.

$$E_{photon} = E_{high} - E_{low} = -13.6\,eV\left(\frac{1}{n_{high}^2} - \frac{1}{n_{low}^2}\right)$$

For $3 \rightarrow 1$: $E_{photon} = -13.6\,eV\left(\frac{1}{3^2} - \frac{1}{1^2}\right) = \boxed{12.089\,eV}$

$$\lambda = \frac{hc}{E} = \frac{1240\,eV\cdot nm}{12.089\,eV} = \boxed{102.6\,nm}$$

For $3 \rightarrow 2$: $E_{photon} = -13.6\,eV\left(\frac{1}{3^2} - \frac{1}{2^2}\right) = \boxed{1.889\,eV}$

$$\lambda = \frac{hc}{E} = \frac{1240\,eV\cdot nm}{1.889\,eV} = \boxed{656.5\,nm}$$

For $2 \rightarrow 1$: $E_{photon} = -13.6\,eV\left(\frac{1}{2^2} - \frac{1}{1^2}\right) = \boxed{10.2\,eV}$

$$\lambda = \frac{hc}{E} = \frac{1240\,eV\cdot nm}{10.2\,eV} = \boxed{121.6\,nm}$$

b. In copper, each atom has an outer electron that is very loosely bound. Thus copper contains a collection of essentially free electrons, with each electron confined to a region about 2.3×10^{-10} across. Estimate the minimum velocity of an electron in copper.

$$\Delta x\,\Delta p \approx \frac{h}{2\pi} \quad \text{with} \quad \Delta p = m\Delta v \simeq V_{min}$$

So $(\Delta x)(mV_{min}) \approx \frac{h}{2\pi}$

$$V_{min} \simeq \frac{h}{2\pi m \Delta x}$$

$$V_{min} \simeq \frac{6.63\times10^{-34}\,J\cdot sec}{2\pi(9.11\times10^{-31}\,kg)(2.3\times10^{-10}\,m)}$$

$$\boxed{V_{min} \approx 5.04\times10^{5}\,\frac{m}{sec}}$$

7. a. The initial activity of a sample of radioactive ^{131}I (iodine) is 2.0×10^{12} Bq. The half-life of ^{131}I is 8.04 days. How many ^{131}I nuclei remain after 21 days?

$$N_0 = \frac{R_0}{\lambda} \quad \text{where } \lambda = \frac{\ln 2}{\tau_{1/2}} = \frac{\ln 2}{(8.04\,d)\left(\frac{24\,h}{1\,d}\right)\left(\frac{3600\,sec}{1\,h}\right)}$$

$$\lambda = 9.98 \times 10^{-7}\ \frac{1}{sec}$$

$$\text{So } N_0 = \frac{2\times 10^{12}\ \frac{1}{sec}}{9.98\times 10^{-7}\ \frac{1}{sec}} = 2\times 10^{18}$$

Thus after 21 days, the number of nuclei is

$$N = N_0 e^{-\lambda t}$$

$$= (2\times 10^{18})\, e^{-\left(\frac{\ln 2}{8.04\,d}\right)(21\,d)}$$

$$= \boxed{3.27\times 10^{17}}$$

b. One of the nuclear fusion reactions that release energy in the Sun is

$$^3He + {}^3He \to {}^4He + {}^1H + {}^1H$$

Find the energy (in MeV) released by this reaction. The mass of ^3He is 3.016029 u, the mass of ^4He is 4.002602 u, and the mass of ^1H is 1.007825 u.

$$Q = [2m(^3He) - m(^4He) - 2m(^1H)]c^2$$

$$= [2(3.016029u) - 4.002602u - 2(1.007825u)]c^2$$

$$= 0.013806\, uc^2$$

$$= 0.013806\,(931.5\ MeV)$$

$$= \boxed{12.86\ MeV}$$