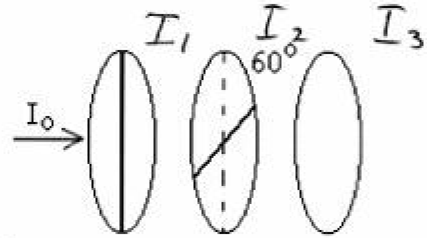
Seat #: __________ Name: KEY

PHYSICS 2200 - EXAM #4 - **SPRING 2009**

1. a. Unpolarized light passes through three polarizing filters. The angle between the polarizing directions of the first and second filter is 60°. The initial intensity of the light is 8 x 10^{-2} W/m^2, and the final intensity of the light (after the third filter) is 5 x 10^{-3} W/m^2. What is the angle between the polarizing directions of the second and third filters?



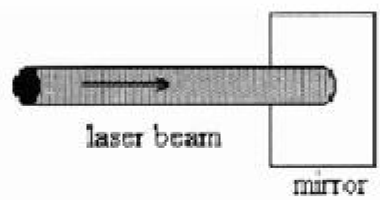
$$I_1 = \tfrac{1}{2} I_0 \ , \quad I_2 = I_1 \cos^2 60^\circ = \tfrac{1}{2} I_0 \cos^2 60^\circ$$

$$I_3 = I_2 \cos^2\theta = \tfrac{1}{2} I_0 \cos^2 60^\circ \cos^2\theta$$

$$\Rightarrow \cos\theta = \sqrt{\frac{2 I_3}{I_0 \cos^2 60^\circ}} = \sqrt{\frac{2(5\times10^{-3}\,W/m^2)}{(8\times10^{-2}\,W/m^2)\cos^2 60^\circ}}$$

$$\cos\theta = 0.7071 \Rightarrow \boxed{\theta = 45^\circ}$$

b. A 5000 W laser beam has a radius of 4 mm. Find the force due to **radiation pressure** on a perfectly reflecting mirror if the beam hits perpendicular to the mirror's surface.



$$I = \frac{\text{power}}{\text{area}} = \frac{5000\,W}{\pi(0.004\,m)^2} = 9.947\times10^{7}\,\frac{W}{m^2}$$

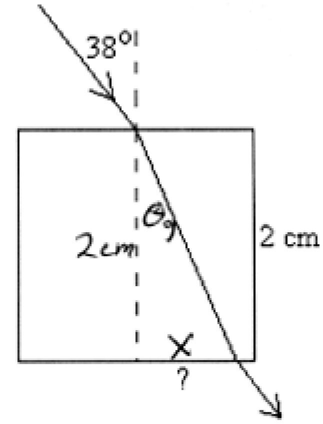
So the force is

$$F = P_{rad} \times \text{area} = \frac{2I}{c} \times \pi r^2$$

$$\Rightarrow F = \frac{2(9.947\times10^{7}\,W/m^2)}{3\times10^{8}\,m/sec} \times \pi(0.004\,m)^2$$

$$= \boxed{3.33\times10^{-5}\,N}$$

2. a. A red laser beam enters the center of a square piece of glass (n = 1.5) at a 38° angle, as shown. If each side of the square is 2 cm, how far from the center of the bottom side does the beam leave the square piece of glass?



$$n_{air} \sin\theta_{air} = n_g \sin\theta_g$$

$$\Rightarrow \sin\theta_g = \frac{n_{air}}{n_g} \sin\theta_{air} = \frac{1}{1.5} \sin 38° = 0.4104$$

$$\Rightarrow \theta_g = 24.23°$$

$$\text{So } x = (2cm)\tan\theta_g = (2cm)\tan(24.23°)$$

$$\boxed{x = 0.9\ cm}$$

b. A candle 8 cm high is placed 40 cm from a converging lens that has a focal length of 60 cm. Find the location and height of the image. Is the image <u>real</u> or <u>virtual</u>?

$$h = 8cm \quad p = 40cm \quad f = 60\ cm$$

$$\frac{1}{f} = \frac{1}{p} + \frac{1}{i} \Rightarrow \frac{1}{i} = \frac{1}{f} - \frac{1}{p} = \frac{p-f}{pf}$$

$$\text{So } i = \frac{pf}{p-f} = \frac{(40cm)(60cm)}{40cm - 60cm} = \boxed{-120\ cm}$$

$$h' = -h\frac{i}{p} = -8cm\left(\frac{-120\ cm}{40\ cm}\right) = \boxed{24cm}$$

This is a $\boxed{\text{virtual image.}}$

3. a. A diffraction grating has 500 lines per centimeter. A laser shines through the grating onto a screen 3 m away. The distance from the central maximum to the second order maximum is 21 cm. Find the wavelength of the laser light.

$$d \sin\theta = d\left(\frac{y}{D}\right) = m\lambda \quad \text{with } m = 2 \text{ and}$$
$$d = \frac{1}{500}\text{ cm} = 2\times10^{-5}\text{ m}$$

$$\text{So } \lambda = \frac{dy}{mD} = \frac{(2\times10^{-5}\text{ m})(0.21\text{ m})}{2(3\text{ m})}$$

$$\lambda = 7\times10^{-7}\text{ m} = \boxed{700\text{ nm}}$$

b. Two blue (λ = 450 nm) billiard balls lie on a beach. What is the smallest distance between the balls that will allow them to be resolved by a spy satellite's telescope with a 0.5 m diameter mirror at an altitude of 820 km?

$$S = r\theta_{min} = r\left(\frac{1.22\lambda}{d}\right)$$

$$S = (820\times10^{3}\text{ m})\left(\frac{1.22(450\times10^{-9}\text{ m})}{0.5\text{ m}}\right)$$

$$\boxed{S = 0.900\text{ m}}$$