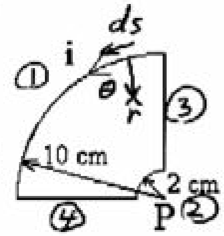
Name: KEY

PHYSICS 2220 - EXAM #3 - SPRING 2009

1. a. A 175 mA current flows through the wire shown. The radius of the larger arc is 10 cm, and the radius of the smaller arc is 2 cm. Use the Biot-Savart law to find the magnetic field at point P. Is the field into or out of the page?



$$B = \frac{\mu_0 i}{4\pi}\int \frac{ds \sin\theta}{r^2} \quad \text{where } \theta = 90° \text{ for ① and ②}$$
$$\theta = 0° \text{ for ④ and } 180° \text{ for ③}$$

Let "out of the page" be positive. Then

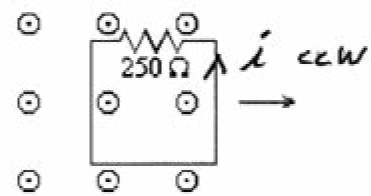
$$B = \frac{\mu_0 i}{4\pi}\frac{1}{r_1^2}\int_{①} ds - \frac{\mu_0 i}{4\pi}\frac{1}{r_2^2}\int_{②} ds$$

$$= \frac{\mu_0 i}{4\pi}\left[\frac{1}{r_1^2}\frac{2\pi r_1}{4} - \frac{1}{r_2^2}\frac{2\pi r_2}{4}\right] = \frac{\mu_0 i}{8}\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$$

$$B = \frac{(4\pi\times10^{-7}\frac{Wb}{A\cdot m})(0.175A)}{8}\left[\frac{1}{0.1m} - \frac{1}{0.02m}\right] = \boxed{-1.10\times10^{-6}T}$$

into the page

b. Each side of a square loop of wire (1 turn) is 50 cm long, and the loop contains a 250 Ω resistor. Initially, the wire is in a region where there is a uniform magnetic field of 0.74 T directed out of the page. The wire is pulled out of the region at constant speed in a time of 0.037 s. Find the current that flows in the wire while it is leaving the magnetic field region. Draw the direction of the current.



$$\mathcal{E} = -N\frac{\Delta(BA\cos\theta)}{\Delta t} = -1\,\frac{B_f A\cos\theta - B_i A\cos\theta}{\Delta t} \quad (\theta = 0°)$$

$$\text{Since } B_f = 0, \quad \mathcal{E} = \frac{B_i A}{\Delta t} = \frac{(0.74T)(.5m)^2}{0.037\text{ sec}} = 5V$$

$$\text{So } i = \frac{\mathcal{E}}{R} = \frac{5V}{250\Omega} = \boxed{2\times10^{-2}A}$$

2. a. A solenoid has a length of 40 cm and has a radius of 3.4 cm. It is wound with 1280 turns of wire. Initially a current of 1.6 A flows through the wire, but the current is smoothly reduced to half of its initial value in 0.005 s. Find the voltage across the solenoid while the current is being reduced.

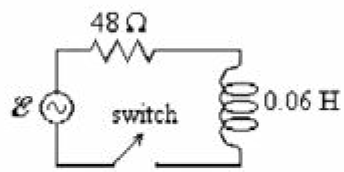
$$L = \mu_0 n^2 A l = \left(4\pi \times 10^{-7} \tfrac{Wb}{A\cdot m}\right)\left(\frac{1280}{0.4m}\right)^2 \pi (0.034m)^2 (0.4m)$$

$$= 0.01869\ H$$

$$\mathcal{E} = -L\frac{di}{dt} = -0.01869\,H\ \frac{0.8A - 1.6A}{0.005s}$$

$$= \boxed{2.99\ V}$$

b. In the circuit shown at right, the switch is closed at t = 0. If the emf of the battery is $\mathcal{E}$ = 120 V, find the time for the current to reach 2.36 amps.



$$i = \frac{\mathcal{E}}{R}\left(1 - e^{-Rt/L}\right)$$

$$\frac{iR}{\mathcal{E}} = 1 - e^{-Rt/L} \quad \text{so} \quad e^{-Rt/L} = 1 - \frac{iR}{\mathcal{E}}$$

$$\Rightarrow -\frac{Rt}{L} = \ln\left(1 - \frac{iR}{\mathcal{E}}\right) \Rightarrow t = -\frac{L}{R}\ln\left(1 - \frac{iR}{\mathcal{E}}\right)$$

$$\text{So}\quad t = -\frac{0.06\,H}{48\,\Omega}\ln\left(1 - \frac{(2.36A)(48\Omega)}{120V}\right)$$

$$= \boxed{3.60 \times 10^{-3}\ sec}$$

3. A series RLC circuit has a 600 Ω resistor, a 0.4 H inductor, and a capacitor. They are connected to an ac power supply with a maximum emf of 18 V.

a. Find the current when the power supply is set to the angular resonance frequency of 280 rad/s.

At resonance, $X_L = X_C$ and $Z = R$

$$I_m = \frac{\mathcal{E}_m}{R} = \frac{18\,V}{600\,\Omega} = \boxed{3\times10^{-2}\,A}$$

b. Find the maximum voltage across the resistor, inductor, and capacitor at the resonance frequency.

$$V_R = I_m R = (3\times10^{-2}\,A)(600\,\Omega) = \boxed{18\,V}$$

$$V_L = I_m \omega_o L = (3\times10^{-2}\,A)(280\,\tfrac{rad}{sec})(0.4\,H)$$
$$= \boxed{3.36\,V}$$

$V_C = I_m \frac{1}{\omega_o C}$ where C comes from $\omega_o = \frac{1}{\sqrt{LC}}$

$$\Rightarrow C = \frac{1}{\omega_o^2 L} = \frac{1}{(280\,\frac{rad}{sec})^2(0.4\,H)} = 3.189\times10^{-5}\,F$$

So $V_C = (3\times10^{-2}\,A)\frac{1}{(3.189\times10^{-5}\,F)(280\,\frac{rad}{sec})} = \boxed{3.36\,V}$

(EASIER WAY! Since $X_L = X_C$ at resonance,
$V_C = V_L = 3.36\,V$)