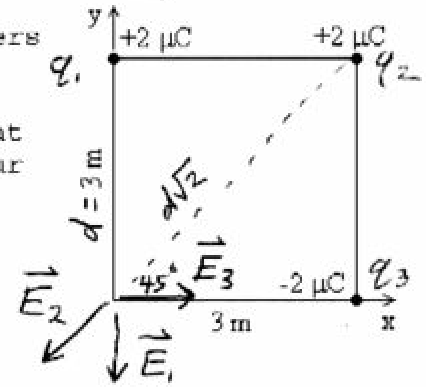
Name: KEY

PHYSICS 2220 - EXAM #1 - SPRING 2009

1. Three charges are placed at the corners of a square, as shown.



a. Find the electric field vector at the origin. You may express your answer either in unit vector notation or as a magnitude and direction.

$$E_1 = E_3 = k\frac{q}{d^2}$$

$$E_2 = k\frac{q}{(d\sqrt{2})^2} = k\frac{q}{2d^2}$$

$$E_x = E_3 - E_2\cos 45^\circ = k\frac{q}{d^2} - k\frac{q}{2d^2}\cos 45^\circ$$
$$= \left(9\times10^9 \frac{Nm^2}{C^2}\right)\frac{(2\times10^{-6}C)}{(3m)^2}\left(1 - \tfrac{1}{2}\cos 45^\circ\right)$$
$$= 1.29\times10^3\ N/C$$

$$E_y = -E_1 - E_2\sin 45^\circ = -k\frac{q}{d^2} - k\frac{q}{2d^2}\sin 45^\circ$$
$$= -\left(9\times10^9 \frac{Nm^2}{C^2}\right)\frac{(2\times10^{-6}C)}{(3m)^2}\left(1 + \tfrac{1}{2}\sin 45^\circ\right)$$
$$= -2.71\times10^3\ N/C$$

$$\boxed{\vec{E} = 1.29\times10^3 \frac{N}{C}\hat{i} - 2.71\times10^3\frac{N}{C}\hat{j}}\ \text{or}\ \boxed{3.00\times10^3 \frac{N}{C}\text{ at } -64.5^\circ \text{ below x-axis}}$$

b. An electron is placed at the origin and released. Find its initial acceleration in either unit vector notation or as a magnitude and direction.

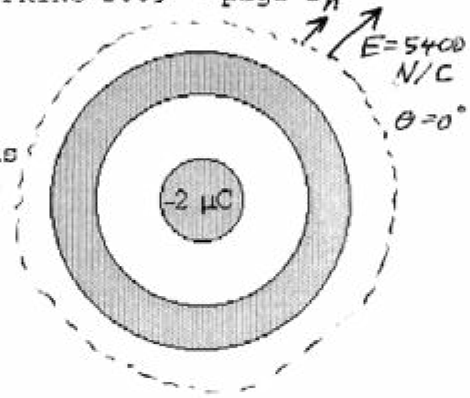
$$\vec{F} = q\vec{E} = m_e\vec{a} \quad \text{so} \quad \vec{a} = \frac{q\vec{E}}{m_e}$$

$$\vec{a} = \frac{(-1.6\times10^{-19}C)(1.29\times10^3\frac{N}{C}\hat{i} - 2.71\times10^3\frac{N}{C}\hat{j})}{9.11\times10^{-31}kg}$$

$$\boxed{\vec{a} = -2.27\times10^{14}\frac{m}{sec^2}\hat{i} + 4.76\times10^{14}\frac{m}{sec^2}\hat{j}}$$

or $$a = \frac{qE}{m_e} = \frac{(1.6\times10^{-19}C)(3\times10^3\ N/C)}{9.11\times10^{-31}Kg}$$

$$\boxed{\vec{a} = 5.27\times10^{14}\frac{m}{sec^2} \text{ at } 115.5^\circ \text{ above x-axis}}$$



2. A solid conducting sphere has a radius of 6 cm. It is at the center of a hollow conducting spherical shell of inner radius 16 cm and outer radius 20 cm. The charge on the central sphere is -2 μC. The electric field 3.3 m from the center is 5400 N/C, directed outward.

a. Use Gauss' law to find the total charge on the hollow conducting shell.

$$\oint E\,dA \underbrace{\cos\theta}_{=1} = \frac{q_{enc}}{\varepsilon_0} \quad \text{with } \theta = 0^\circ$$

$$E\int dA = \frac{q_{enc}}{\varepsilon_0} \quad \text{or} \quad E\,4\pi r^2 = \frac{q_{enc}}{\varepsilon_0}$$

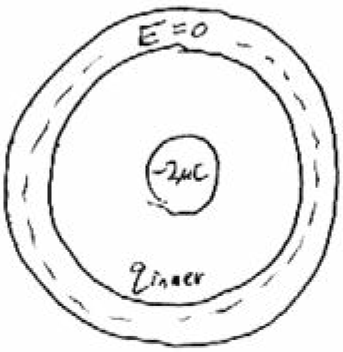
$$q_{enc} = \varepsilon_0 E\,4\pi r^2 = (8.85\times10^{-12}\tfrac{C^2}{Nm^2})(5400\tfrac{N}{C})4\pi(3.3m)^2$$

$$q_{enc} = 6.54\times10^{-6}\,C = 6.54\,\mu C$$

$$\text{Since } q_{enc} = q_{hollow} - 2\mu C = 6.54\,\mu C$$

$$\boxed{q_{hollow} = 8.54\,\mu C}$$

b. Find the charge on the inner and outer surfaces of the hollow conducting shell.



$$\oint E\,dA\cos\theta = \frac{q_{enc}}{\varepsilon_0} = 0 \quad \text{since } \vec{E} = 0$$

$$\text{So} \quad q_{enc} = q_{inner} - 2\mu C = 0$$

$$\Rightarrow \boxed{q_{inner} = 2\mu C}$$

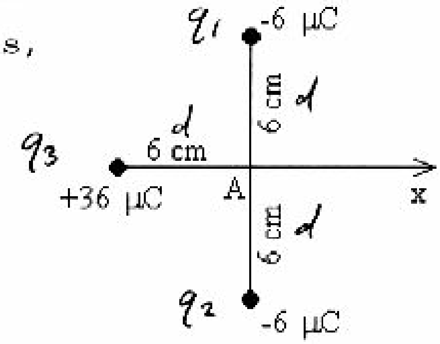
$$\text{But} \quad q_{hollow} = q_{inner} + q_{outer}$$

So

$$q_{outer} = q_{hollow} - q_{inner} = 8.54\,\mu C - 2\mu C$$

$$\boxed{q_{outer} = 6.54\,\mu C}$$

3. The figure at right shows three charges, one of +36 μC and two of -6 μC.



a. Find the electric potential (voltage) at point A, assuming that V = 0 at infinity.

$$V_A = k\frac{q_1}{d} + k\frac{q_2}{d} + k\frac{q_3}{d}$$

$$= \frac{k}{d}(q_1 + q_2 + q_3)$$

$$= \frac{(9\times10^{9}\frac{Nm^2}{C^2})}{0.06\ m}(-6-6+36)\times10^{-6}C$$

$$V_A = \boxed{3.6\times10^{6}\ V}$$

b. A small glass sphere has a mass of 48 grams and has a charge of +6 μC. It is released at rest at point A. Use conservation of energy to find the sphere's speed when it has traveled a great distance in the positive x-direction, out to where V is practically zero.

$$K_i + U_i = K_f + U_f \quad \text{where } K_i = 0$$

$$K_f = U_i - U_f = q(V_i - V_f)$$

$$\frac{1}{2}mv^2 = q(V_A - V_\infty) \quad \text{with } V_\infty = 0$$

$$\text{Thus } \frac{1}{2}mv^2 = qV_A$$

$$v = \sqrt{\frac{2qV_A}{m}}$$

$$= \sqrt{\frac{2(6\times10^{-6}C)(3.6\times10^{6}V)}{0.048\ kg}}$$

$$= \boxed{30.0\ \frac{m}{sec}}$$