

Physics 4610 Quantum mechanics
Quiz 6

NAME:

Consider the wavefunction $\psi(\theta)$ with $-\pi \leq \theta \leq \pi$. If the wavefunction satisfies

$$\psi(\pi) = \psi(-\pi),$$

show that the operator

$$L = \frac{\hbar}{i} \frac{d}{d\theta}$$

is a hermitian operator.

$$\langle \psi(\theta) | L \psi(\theta) \rangle = \int_{-\pi}^{\pi} \psi^* \frac{\hbar}{i} \frac{d}{d\theta} \psi d\theta$$

Integration by Parts: $u = \psi^* \quad dv = \frac{d\psi}{d\theta} d\theta$

$$= \frac{\hbar}{i} \left[\psi^* \psi \right]_{-\pi}^{\pi} - \frac{\hbar}{i} \int_{-\pi}^{\pi} \psi \frac{d\psi^*}{d\theta} d\theta$$

$$= -\frac{\hbar}{i} \int_{-\pi}^{\pi} \frac{d\psi^*}{d\theta} \psi d\theta = \int_{-\pi}^{\pi} \left(\frac{\hbar}{i} \frac{d\psi}{d\theta} \right)^* \psi d\theta = \langle L\psi | \psi \rangle$$

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