2.4

nth stationary state $= \sqrt{\frac{2}{a}} \sin\frac{n\pi}{a}x = \psi_n(x)$

$$\langle x \rangle = \int \psi_n^*(x)\, x\, \psi_n(x)\, dx$$

$$= \frac{2}{a}\int_0^a x \sin^2\frac{n\pi}{a}x\, dx = \boxed{\frac{a}{2}}$$

$$\langle x^2 \rangle = \int \psi_n^*\, x^2\, \psi_n\, dx$$

$$= \frac{2}{a}\int_0^a x^2 \sin^2\frac{n\pi}{a}x\, dx = \cdots$$

$$\langle x^2 \rangle = \boxed{a^2\left(\frac{1}{3} - \frac{1}{2(n\pi)^2}\right)}$$

$$\langle p \rangle = \int \psi_n^* \frac{\hbar}{i}\frac{\partial}{\partial x}\psi_n(x)\, dx$$

$$= \frac{2}{a}\cdot\frac{\hbar}{i}\int_0^a \left(\sin\frac{n\pi}{a}x\right)\left(\frac{n\pi}{a}\cos\frac{n\pi}{a}x\right)dx$$

$$\langle p \rangle = \boxed{0}$$

2.4 Continues

$$\langle p^2 \rangle = \int \psi_n^*(x) \left(\frac{\hbar}{i}\right)^2 \frac{\partial^2}{\partial x^2} \psi_n(x)\, dx$$

$$= -\hbar^2 \cdot \frac{2}{a} \int_0^a \left(\sin \frac{n\pi}{a} x\right) \frac{\partial^2}{\partial x^2} \left(\sin \frac{n\pi}{a} x\right) dx$$

$$= \cdots = \boxed{\left(\frac{n\pi}{a}\hbar\right)^2}$$

$$\sigma_x^2 = \langle x^2 \rangle - \langle x \rangle^2 \Rightarrow \sigma_x = \frac{a}{2}\sqrt{\frac{1}{3} - \frac{2}{n^2\pi^2}}$$

$$\sigma_p^2 = \langle p^2 \rangle - \langle p \rangle^2 \Rightarrow \sigma_p = \frac{n\pi\hbar}{a}$$

$$\sigma_x \sigma_p = \frac{\hbar}{2}\sqrt{\frac{n^2\pi^2}{3} - 2}$$

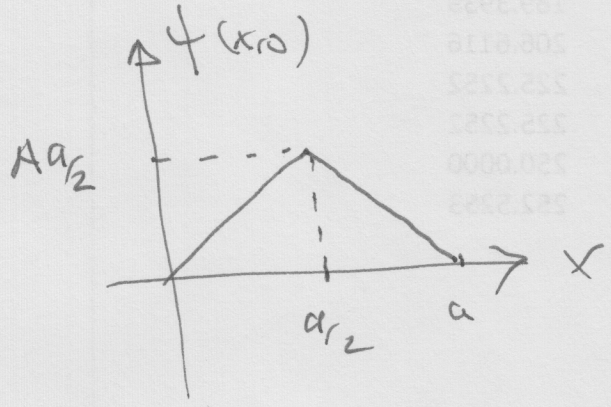
for $n = 1$ $\sigma_x \sigma_p$ is the smallest.

$$\sigma_x \sigma_p \Big|_{n=1} = 1.136\, \hbar/2 > \hbar/2$$

2.8

$$\psi(x,0) = \begin{cases} Ax & 0 \le x \le a/2 \\ A(a-x) & a/2 \le x \le a \end{cases}$$

a)



Normalization:

$$A^2\left[\int_0^{a/2} x^2\,dx + \int_{a/2}^{a} (a-x)^2\,dx\right] = 1$$

$$\boxed{A = \frac{2\sqrt{3}}{\sqrt{a^3}}}$$

b) To find $\psi(x,t)$, we need to expand $\psi(x,0)$ in terms of the stationary states of infinite square well.

2.> Continues

$$\psi(x,0) = \sum_n c_n \psi_n(x)$$

$$c_n = \int \psi(x,0)\, \psi_n(x)\, dx$$

Thus, we need $c_n = ?$

$$c_n = \sqrt{\frac{2}{a}} A \left\{ \int_0^{a/2} x \sin\frac{n\pi}{a}x\, dx + \int_{a/2}^{a} (a-x) \sin\frac{n\pi}{a}x\, dx \right\}$$

$$= \dots = \begin{cases} 0 & n = \text{even} \\ (-1)^{\frac{n-1}{2}} \dfrac{4\sqrt{6}}{n^2\pi^2} & n = \text{odd} \end{cases}$$

$$\Rightarrow \psi(x,t) = \frac{4\sqrt{6}}{\pi^2}\sqrt{\frac{2}{a}} \sum_{n=1,3,5,\dots} (-1)^{\frac{n-1}{2}} \frac{1}{n^2} \sin\frac{n\pi}{a}x\, e^{-iE_n t/\hbar}$$

c) Prob. of finding in state $\psi_1 = P_1$

$$= |c_1|^2 = \frac{(16)(6)}{\pi^4} = 0.985$$

d) $\langle H \rangle = \sum_n |c_n|^2 E_n =$

→ Continues

2.) Continues

$$\langle H \rangle = \frac{96}{\pi^4} \frac{\pi^2 \hbar^2}{2ma^2} \underbrace{\left(\frac{1}{1} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots \right)}_{\pi^2/8}$$

$$\langle H \rangle = \frac{48 \hbar^2}{\pi^2 ma^2} \cdot \frac{\pi^2}{8} = \boxed{\frac{6\hbar^2}{ma^2}}$$

2.8

a) $$\psi(x,0) = \begin{cases} A & 0 \le x \le a/2 \\ 0 & \text{otherwise} \end{cases}$$

Normalization:

$$\int |\psi(x,0)|^2 dx = 1 \Rightarrow$$

$$\int_0^{a/2} A^2 dx + 0 = 1 \Rightarrow$$

$$\boxed{A = \sqrt{\frac{2}{a}}}$$

$$\psi(x,0) = \sqrt{\frac{2}{a}}$$

b) For $E = \frac{\pi^2 \hbar^2}{2ma^2}$ means that

Particle is in state $n=1$:

$$\text{Prob} = |c_1|^2$$

We need $c_1 = ?$.

2.8 Continues

$$c_1 = \int \psi(x,0)\,\psi_1(x)\,dx$$

$$= \int_0^{a/2} \sqrt{\frac{2}{a}} \cdot \sqrt{\frac{2}{a}} \sin\frac{\pi}{a}x \, dx$$

$$\boxed{c_1 = \frac{2}{\pi}}$$

$$P_1 = |c_1|^2 = \left(\frac{2}{\pi}\right)^2 = 0.405$$