Problems of chapter 1

1.3

$$\rho(x) = A e^{-\lambda(x-a)^2}$$

a) $$\int_{-\infty}^{+\infty} \rho(x)\,dx = 1 \Rightarrow$$

$$A \int_{-\infty}^{+\infty} e^{-\lambda(x-a)^2} dx = 1$$

take $x - a \equiv z \rightarrow dx = dz$

$$A \underbrace{\int_{-\infty}^{+\infty} e^{-\lambda z^2} dz}_{\sqrt{\frac{\pi}{\lambda}}} = 1 \Rightarrow A\sqrt{\frac{\pi}{\lambda}} = 1 \rightarrow$$

$$\boxed{A = \sqrt{\frac{\lambda}{\pi}}}$$

b) $$\langle x \rangle = \sqrt{\frac{\lambda}{\pi}} \int x\, e^{-\lambda(x-a)^2} dx$$

using the same substitute: $z \equiv x - a$

$$\langle x \rangle = \sqrt{\frac{\lambda}{\pi}} \left[\int_{-\infty}^{+\infty} z\, e^{-\lambda z^2} dz + a \int_{-\infty}^{+\infty} e^{-\lambda z^2} dz \right]$$

1.3 Continues

The first integral is "odd" and is zero.

$$\langle x \rangle = \sqrt{\frac{\lambda}{\pi}} \left[a \sqrt{\frac{\pi}{\lambda}} \right] = a$$

$$\boxed{\langle x \rangle = a}$$

$$\langle x^2 \rangle = \sqrt{\frac{\lambda}{\pi}} \int_{-\infty}^{+\infty} x^2 \, e^{-\lambda (x-a)^2} \, dx$$

again $z \equiv x - a \rightarrow z^2 = (x-a)^2$

$$dz = dx$$

$$x^2 = z^2 + a^2 + 2za$$

$$\langle x^2 \rangle = \sqrt{\frac{\lambda}{\pi}} \left[\int_{-\infty}^{+\infty} z^2 e^{-\lambda z^2} dz + a^2 \int_{-\infty}^{+\infty} e^{-\lambda z^2} dz + 2a \int_{-\infty}^{+\infty} z e^{-\lambda z^2} dz \right]$$

$$\langle x^2 \rangle = \sqrt{\frac{\lambda}{\pi}} \left[\frac{1}{2\lambda} \sqrt{\frac{\pi}{\lambda}} + a^2 \sqrt{\frac{\pi}{\lambda}} + 0 \right]$$

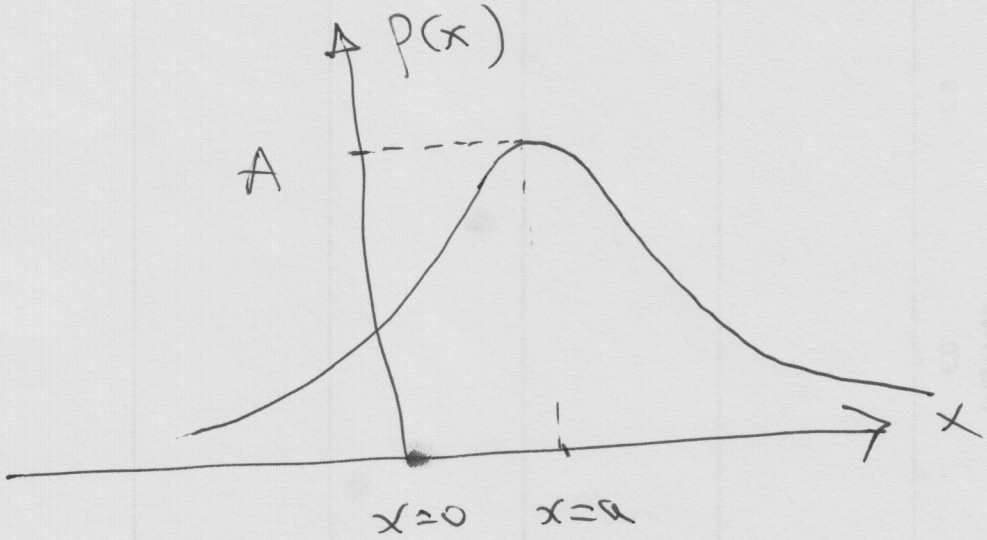
$$\boxed{\langle x^2 \rangle = a^2 + \frac{1}{2\lambda}}$$

1.3 Continues

$$\sigma^2 = \langle x^2 \rangle - \langle x \rangle^2$$

$$= \left(a^2 + \frac{1}{2\lambda}\right) - (a)^2 = \boxed{\frac{1}{2\lambda}}$$

c)



1.4

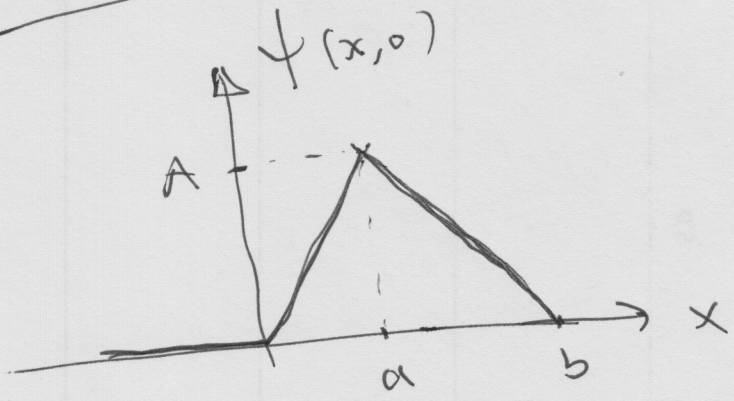
$$\psi(x,0) = \begin{cases} A\frac{x}{a} & 0 \le x \le a \\ A\frac{b-x}{b-a} & a \le x \le b \\ 0 & \text{otherwise} \end{cases}$$

a) $$\int_{-\infty}^{+\infty} |\psi(x,0)|^2 \, dx = 1 \Rightarrow$$

$$A^2 \left[\int_0^a \frac{x^2}{a^2} dx + \int_a^b \left(\frac{b-x}{b-a}\right)^2 dx \right] = 1$$

$$\Rightarrow \boxed{A = \sqrt{\frac{3}{b}}}$$

b)



c) $P(x)$ is max ~~[illegible]~~ at $\boxed{x = a}$

1.4 Continues

d) $$P_{\text{left of } a} = \int_0^a |\psi|^2 dx$$

$$= A^2 \int_0^a \frac{x^2}{a^2} dx = \frac{3}{ba^2} \int_0^a x^2 dx$$

$$\boxed{P_{\text{left of } a} = \frac{a}{b}}$$

If $a = b$ $\quad P_{\text{left of } a} \to 1$

If $a = b/2$ $\quad P_{\text{left of } a} \to 1/2$

e) $$\langle x \rangle = \int x |\psi|^2 dx =$$

$$\frac{3}{b}\left[\int_0^a x \frac{x^2}{a^2} dx + \int_a^b x \frac{(b-x)^2}{(b-a)^2} dx \right]$$

$$= \cdots = \boxed{\frac{2a+b}{4}}$$

1.5

$$\psi(x,t) = A e^{-\lambda|x|} e^{-i\omega t}$$

a) $\int |\psi(x,t)|^2 dx = 1$

$$A^2 \int_{-\infty}^{+\infty} e^{-2\lambda|x|} dx = 1$$

$$= 2\int_0^{\infty} e^{-2\lambda x} dx = 2\left(\frac{e^{-2\lambda x}}{-2\lambda}\right)_0^{\infty}$$

$$\Rightarrow \boxed{A = \sqrt{\lambda}}$$

b) $\langle x \rangle = \int x |\psi|^2 dx$

$$= A^2 \int_{-\infty}^{+\infty} x\, e^{-2\lambda|x|} dx \longrightarrow 0 \qquad \boxed{\langle x \rangle = 0}$$

odd funct.

$$\langle x^2 \rangle = A^2 \int_{-\infty}^{+\infty} x^2 e^{-2\lambda|x|} dx \Rightarrow$$

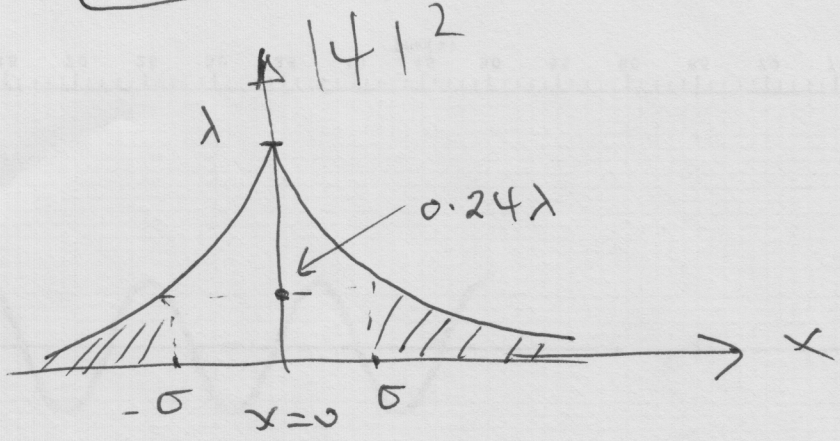
even funct.

$$\langle x^2 \rangle = 2A^2 \int_0^{\infty} x^2 e^{-2\lambda x} dx = \boxed{\frac{1}{2\lambda^2}}$$

1.5

c) $\sigma^2 = \langle x^2 \rangle - \langle x \rangle^2 \Rightarrow$

$$\boxed{\sigma = \frac{1}{\sqrt{2}\lambda}}$$



$$|\psi|^2 \text{ at } x = \pm\sigma \quad = A^2 e^{-2\lambda\sigma}$$

$$= \lambda e^{-2\lambda \frac{1}{\sqrt{2}\lambda}} = \lambda e^{-\sqrt{2}} = 0.24\lambda$$

$$\text{Prob. outside} = 2 \int_{\sigma}^{\infty} |\psi|^2 dx$$

$$= 2A^2 \int_{\sigma}^{\infty} e^{-2\lambda x} dx = \cdots = \boxed{0.24}$$

or 24%