Wave Optics Study Guide

For constructive interference with two or more slits (or a diffraction grating),

$$d \sin \theta = m \lambda.$$ 

You should understand why this formula is correct and be able to derive it using an appropriate diagram. You should be able to draw the diffraction pattern for two slits or a grating.

You should be able to determine which wavelengths of light will interfere constructively and destructively at a thin film. This entails understanding both the difference in path lengths for rays reflecting off the front and back surfaces and also any possible phase changes upon reflection (which depend on whether the material surrounding the film has a higher or lower index of refraction).

You should understand and be able to draw the diffraction pattern produced by a single slit. The formula for the angles of destructive interference is

$$a \sin \theta = m \lambda.$$ 

You should be able to derive this formula using an appropriate diagram.

Quantum Mechanics Study Guide

Basic formulas:

$$E = hf$$ (Einstein); \hspace{1cm} $$p = h/\lambda$$ (de Broglie).

The first relation is useful only for photons (for which $$E = pc$$). To relate the wavelength of some other particle to its energy, use $$K = p^2/2m$$. Don’t forget the numerical value $$h = 6.63 \times 10^{-34} \text{ J-s}$$.

You should be able to explain the experimental evidence that light behaves like particles (photonic-electric effect) and electrons behave like waves (diffraction and interference).

You should understand the concept of a “wavefunction”, and be able to draw wavefunctions with definite position and definite momentum. You should understand the interpretation of the square of the wavefunction as being proportional to the probability of finding the particle at a given location.

You should be able to draw the definite-energy wavefunctions for a particle in a one-dimensional “box”, and explain why only certain energies are allowed. You should know the energy level formulas

$$E_n = \frac{h^2 n^2}{8mL^2}$$ \hspace{1cm} (1-D box), \hspace{1cm} $$E_n = -\frac{13.6 \text{ eV}}{n^2}$$ \hspace{1cm} (H atom),

and be able to derive the former (but not the latter). You should understand why, and under what conditions, energy is quantized in quantum mechanics.

You should understand the concept of transitions between energy levels, and be able to compute the photon energies and wavelengths for transitions between levels for any system whose energy levels you know.
Practice Problems

These problems are taken from tests that I gave in this course in previous years. They should be good practice, and give a good picture of the level of difficulty of our upcoming test. Be warned, however, that they do not comprehensively cover all the material that may be on this test.

1. Light from a laser is sent through a diffraction grating to form a pattern of five bright spots. Suppose that the spots are numbered 1 to 5 from left to right. Spot 3 is straight ahead, in the direction of the incident beam.

   (a) If the angle between spots 3 and 4 is 20°, what is the angle between spots 4 and 5? (Please show your work.)

   (b) If the wavelength of the laser light were doubled, how would the pattern change? (Please be as precise as you can.)

2. A 400-nanometer-thick layer of oil ($n = 1.25$) lies on top of a pool of water ($n = 1.33$). Sunlight strikes the surface from directly above, producing a colored reflection.

   (a) What wavelength(s) of visible light (as measured in a vacuum) are not reflected by the layer of oil? Please explain your reasoning, justifying any formulas you use.

   (b) To the human eye, roughly what would be the color of the reflected light?

3. In a photoelectric cell, blue light (with a wavelength of 450 nm) strikes a metal plate (the cathode), ejecting electrons, some of which accumulate on the anode. An ideal voltmeter, which draws no current, measures the voltage difference between the electrodes.

   (a) Soon after the experiment begins, the voltmeter stabilizes at a fixed value and remains there, as long as the wavelength and intensity of the light do not change. Suppose now that the intensity of the light is increased. What happens to the reading on the meter?

   (b) Does the observation you described in part (a) support the idea that light is a wave, or a particle, or both, or neither? Explain briefly. (If you are unsure of your answer to part (a), discuss the implications of whatever answer you believe is most likely.)

   (c) How could you use an apparatus like this to measure the value of Planck’s constant, $h$?

4. One method of studying the structure of a crystal is to aim a beam of x-rays into it and observe the resulting diffraction pattern. Ideally, the wavelength of the x-rays should be about the same as the spacing between rows of atoms, typically $10^{-10}$ meters. What is the energy of an x-ray photon with this wavelength?

5. Another method of studying crystal structure is neutron diffraction, in which a beam of neutrons is aimed at the crystal and the resulting diffraction pattern is observed. Calculate the kinetic energy of a neutron whose wavelength is $10^{-10}$ meters. (The mass of a neutron is $1.67 \times 10^{-27}$ kg.)

6. In quantum mechanics, the “state” of an electron is described by a “wavefunction”, $\psi(x)$. Explain, in your own words, why it is impossible for an electron to simultaneously have both a well-defined position and a well-defined momentum. (You may find it helpful to draw some examples of wavefunctions.)

7. A certain type of atom has four allowed energy levels, at 1.0 eV, 3.0 eV, 5.0 eV, and 7.0 eV. Draw an energy-level diagram for this atom, and calculate the wavelengths of all of the colors of light (visible and invisible) that it could conceivably emit. Be sure that your logic is clear, and explain briefly how you determined that no other wavelengths are possible.