Problem Set 9
(due Friday, October 28)

1. (a) How long does it take a radio signal to travel 150 km (perhaps from a commercial transmitter to your car stereo)? (b) We see a full moon by reflected sunlight. How long does it take the light to get from the sun to our eyes? (Look up the data you need inside the front cover of your textbook.) (c) What is the round-trip travel time for a radio signal going from earth to a spaceship orbiting Saturn (and back)? (The distance to Saturn is $1.3 \times 10^9$ km.) (d) The Crab Nebula, which is about 6500 light-years distant, is the remnant of a supernova explosion recorded by Chinese astronomers in A.D. 1054. When did the explosion actually take place?

2. Initially unpolarized light, traveling in the $z$ direction, is sent through two ideal polarizers in succession and, when it emerges, is polarized at $20^\circ$ from the $y$ axis and has half its initial intensity. What are the orientations of the two polarizers?

3. Suppose that you have a beam of polarized light and you want to rotate its direction of polarization by $90^\circ$, by sending it through one or more polarizers. (a) What is the minimum number of polarizers required? (b) What is the minimum number of polarizers required if the transmitted intensity must be no less than 60% of the initial intensity?

4. Two flat mirrors are joined at a $90^\circ$ angle. A ray of light, traveling in the plane perpendicular to both mirrors, reflects first off one of them and then off the other. Prove that it returns in the same direction that it came from. (This is the principle of a “corner reflector”, used to reflect light back to its source no matter what direction the source is in.)

5. The wavelength of yellow sodium light in air is 589 nm. (a) What is its frequency? (b) What is its wavelength in a piece of glass with an index of refraction of 1.52? (c) What is the speed of light in this piece of glass?

6. An underwater scuba diver sees the sun at an apparent angle of $50^\circ$ from the horizontal. Where is the sun?

7. A light ray initially in water enters a transparent substance at an angle of incidence of $37^\circ$, and the transmitted ray bends toward the normal by $10^\circ$ (relative to the initial direction of travel). What is the index of refraction of the substance?

8. An optical fiber is made of a clear plastic for which the index of refraction is 1.50. It is encased in another type of plastic with an index of 1.30. For what angles with the surface does light remain contained within the fiber?
9. A ray of white light makes an angle of incidence of $35^\circ$ on one face of a prism of fused quartz; the prism’s cross-section is an equilateral triangle. Sketch the light as it passes through the prism and out the other side, showing the paths of (a) blue light, (b) yellow-green light, and (c) red light. (See page 10 of the Refraction chapter of your lab manual for needed data.)

10. You are looking straight downward at a penny that lies at the bottom of a pool of liquid of depth $d$ and index of refraction $n$. By considering two or more rays of light coming up to your eye from the penny, show that its apparent depth is $d' = d/n$. (Hint: Since the rays are nearly vertical, you can use the small-angle approximation $\sin \theta \approx \tan \theta$.)

11. The figure below shows an object (represented by an arrow) and a convex mirror. The center of curvature of the mirror is marked $C$. Using a graphical construction similar to the one done in class, carefully determine the position of the image formed by the mirror. Do not use any equations, or even what you may know about the “focal length”. Is the image real or virtual?

12. Suppose an object is placed in front of a concave spherical mirror, at a distance from the mirror that is closer than the focal length. Use a ray diagram to show that the image of this object is right-side up and “virtual”.

13. The figure below shows a converging lens, its two focal points, and an object. Use a carefully constructed ray diagram to determine the position and size of the image. Is the image real or virtual?
14. The figure below shows a diverging lens, its two focal points, and an object. Use a carefully constructed ray diagram to determine the position and size of the image. Is the image real or virtual?

The ether trembled at his agitations
In a manner so familiar that I only need to say,
In accordance with Clerk Maxwell’s six equations
It tickled peoples’ optics far away.

—A. A. Robb
Study Guide

You should have a general understanding of EM waves, including: the approximate wavelength range of visible light; which colors have longer/shorter wavelengths; the names assigned to invisible wavelengths (infrared, ultraviolet, etc.), and which are longer/shorter than others; the numerical value of the speed of light \((3 \times 10^8 \text{ m/s})\); how EM waves are produced by accelerated charged particles; and how the electric and magnetic fields are arranged within a wave (perpendicular to each other and to the direction of propagation).

You should understand the various types of polarization (vertical, horizontal, diagonal, circular, and random). An ideal polarizing filter transmits linearly polarized light, polarized along its “transmission axis”. When randomly polarized light enters such a filter, half the energy is transmitted. When linearly polarized light enters such a filter, the electric field of the transmitted light is reduced in magnitude by a factor of \(\cos \theta\), where \(\theta\) is the angle between the incoming polarization and the transmission axis. Because the energy of an EM wave is proportional to the square of the field strength, the polarizer reduces the energy by a factor of \(\cos^2 \theta\).

When light hits a smooth surface, it can reflect (bounce off) and/or refract (pass through, usually with some bending at the surface). The angles of incidence \((\theta_1)\), reflection \((\theta'_1)\), and refraction \((\theta_2)\) are all, by convention, measured with respect to the “normal” line, drawn perpendicular to the surface. The law of reflection is \(\theta'_1 = \theta_1\). The law of refraction (Snell’s law) is

\[
n_1 \sin \theta_1 = n_2 \sin \theta_2,
\]

where \(n_1\) is the index of refraction of the first material and \(n_2\) is the index of refraction of the second material. The index of refraction of a vacuum is 1, while the index of air is only slightly larger. Materials like glass have an index of roughly 1.5. The index of refraction is always greater than 1.

If \(n_1\) is greater than \(n_2\), Snell’s law can yield a nonsensical value (greater than 1) for the sine of the angle of refraction. In such cases there is no refracted ray and all the light reflects off the surface (“total internal reflection”).

The index of refraction is inversely related to the speed of light in that material:

\[
n = \frac{\text{speed of light in vacuum}}{\text{speed of light in material}}
\]

The index can be slightly different for different wavelengths (i.e., different colors) of light, so when a beam of white light (a mixture of many colors) enters a material it may be noticeably split into its component colors. This phenomenon is called “dispersion”.

You should be able to draw accurate ray diagrams for flat and spherical mirrors, locating the image of any object that is viewed in the mirror and determining the size of the image. You should understand the difference between a “real” and a “virtual” image. You should also understand the concept of a focal point. You need not memorize any formulas from Chapter 35.

You should understand what the focal point of a lens is, for both converging and diverging lenses. Given the focal length of a lens and the location of an object, you should be able to draw an accurate ray diagram to determine the position and size of the image.