Problem Set 3
(due Thursday, Sept. 9)

1. Suppose that in a particular lightning flash, the voltage difference between a cloud and the ground is one billion volts and the quantity of charge transferred is 30 C. (a) What is the change in energy of that transferred charge? (b) If all this energy could be used to accelerate a 1000 kg car from rest, what would be the car’s final speed?

2. In a Van de Graaff accelerator a proton is accelerated through a voltage difference of 14 MV (megavolts). Assuming that the proton starts from rest, calculate its final kinetic energy, first in joules and then in electron-volts. (1 eV equals $1.6 \times 10^{-19}$ J.)

3. In the illustration below, the horizontal lines represent electric field lines and the vertical lines represent equipotential surfaces. (a) Does the voltage (potential) increase toward the right or toward the left? How can you tell? (b) The voltage at the rightmost equipotential surface is $-100$ V, and the adjacent surfaces differ by 10 V. What is the voltage at the leftmost surface? (c) Suppose that you move an electron from left to right through this region, in such a way that it is always moving very slowly. Is the work that you do on the electron positive or negative? (d) In the same situation, is the work done on the electron by the electric field positive or negative?

4. Consider a positively charged conducting sphere, in static equilibrium. The charge is uniformly distributed over the surface, and the electric field inside is zero. Does this imply that the voltage inside is zero? Explain carefully.

5. Two large, horizontal, parallel conducting plates are 12 cm apart and have charges of equal magnitude and opposite sign on their facing surfaces. An downward electrostatic force of $3.9 \times 10^{-15}$ N acts on an electron placed anywhere between the two plates (except near the edges). (a) Find the electric field at the position of the electron (magnitude and direction). (b) What is the voltage difference between the two plates? Which is at the higher voltage?

6. In a certain region of space there is a uniform electric field of 5900 V/m, pointing in the +x direction. (a) Suppose we take $V = 0$ at the origin. Where else does $V = 0$? (b) Where does $V = 100$ V? (c) Where does $V = -100$ V?
7. The illustration below shows a region in which the electric field strength is 2000 V/m. The illustration is shown actual size, so you can measure distances on it with a ruler. (a) Calculate the dot product \( \vec{E} \cdot \Delta \vec{r} \) for each of the three paths AC, CB, and AB. (b) If the voltage at point A is zero, what is the voltage at points B and C?

![Diagram of electric field paths](image_link)

8. (a) Referring again the illustration above, calculate the electrostatic force felt by a point charge of \(-15 \text{ nC}\) located in this region. (b) From your answer to part (a), calculate the work you would have to do to slowly push this charge from A to B. (c) From your answer to part (b), calculate the voltage difference between points A and B.

9. Consider a region of space in which the electric field everywhere points in the \(+x\) direction. In the region where \(x < 0\), the magnitude of the field is 1000 V/m, but in the region where \(x > 0\), the magnitude of the field is 3000 V/m. (a) Sketch this electric field. (b) Consider a small particle whose charge is 1 nC. You wish to move this particle backwards along the \(x\)-axis from the point \(x = 2 \text{ m}\) to the point \(x = -2 \text{ m}\). Calculate the work required for this operation, by determining the electrostatic force on the particle in each region and multiplying by the displacement. (c) Use your answer to part (b) and the definition of voltage to calculate the voltage difference between the starting and ending points.

10. In a repeat of the historic Millikan oil drop experiment, you use a pair of plates measuring 6 cm-square, separated by a distance of 1 cm. By observing the rate at which a particular oil drop falls (acted upon by gravity and air resistance), you determine that its mass is \(8.1 \times 10^{-14} \text{ kg}\). You then turn on the voltage, and note that this droplet is suspended motionless when the voltage between the plates is 5000 V. What is the charge on the droplet? How many fundamental units of charge is this?

11. The figure on the following page shows a CRT oscilloscope tube. The voltage difference between the cathode and anode, which accelerates the electrons, is 1000 V. To the right of the anode are two deflection plates, measuring 3 cm along the direction of the beam and separated by a distance of .5 cm. You wish to use the oscilloscope as a voltmeter, so you connect these plates (from points A and B) to an unknown voltage source, and see that the electron beam is deflected by an angle of 5°. What is the voltage difference between the plates? (Hints: You may assume that the horizontal velocity of the electrons is constant after they pass through the anode. Treat the motion between the plates as a projectile motion problem, but with gravity replaced by the electrostatic force. Express the deflection angle in terms of the components of the final velocity of the electrons.)
12. Suppose you have a roll of aluminum foil and a roll of wax paper. Estimate, *very roughly*, how large a capacitor you could make by sandwitching them together. You may neglect the effect of the wax paper, except as a separator between sheets of foil.

13. Suppose that you have two capacitors, one with known capacitance (such as a parallel-plate capacitor) and one with unknown capacitance. You also have a battery and a working voltmeter. You can determine the unknown capacitance by hooking all three components in series, as shown below. You measure a voltage difference of 11.45 V between points $A$ and $B$, and .55 V between points $B$ and $C$. If the known capacitor has a capacitance of 160 pF, what is the capacitance of the other capacitor?
Voltage (or “electric potential”) is defined as electrostatic energy (of a hypothetical test charge in some given environment) per unit charge:

\[ V(x, y, z) = \frac{U_e}{q_0}. \]

Here \( U_e \) is electrostatic potential energy, and \( q_0 \) is the amount of charge on the test charge. Like the potential energy \( U_e \), the voltage is always relative to some arbitrary reference point (“ground”).

Relation between \( V \) and \( \vec{E} \):

\[ \Delta V = -\vec{E} \cdot \vec{d}. \]

In other words, \( \vec{E} \) points from high voltage to low voltage, and the magnitude \( |\vec{E}| \) is the change in voltage per unit distance as you move in that direction. This relation completely analogous to that between \( U_e \) and \( \vec{F}_e \).

Voltage is a useful quantity because batteries (as well as many grid-connected power supplies) supply a fixed voltage. In a battery, this is because the chemical reaction going on inside supplies a fixed amount of energy to each electron (that is, a fixed energy per unit charge). Voltage differences are also relatively easy to measure, unlike charges and electric field strengths.

In his famous oil drop experiment, Robert Millikan showed that electric charge is quantized, coming only in multiples of \( \pm 1.6 \times 10^{-19} \text{ C} \). An electron carries exactly one such unit of negative charge, while a proton carries exactly one such unit of positive charge.

A capacitor is a device that stores positive charge \( Q \) and negative charge \(-Q\), separated from each other, when a voltage difference \( \Delta V \) is applied. The capacitance is defined as

\[ C = \frac{Q}{\Delta V}, \]

charge per unit voltage. The simplest capacitor consists of two parallel plates separated by a small gap. Using the formula for the field near a plane of charge, you can show that in this case the capacitance is \( \varepsilon_0 A/d \), where \( A \) is the plate area and \( d \) is the separation.