Problems from subsection "Compression of an Ideal Gas" (Section 1.5)

Problem 1.35. Derive equation 1.40 from equation 1.39.

Problem 1.36. In the course of pumping up a bicycle tire, a liter of air at atmospheric pressure is compressed adiabatically to a pressure of 7 atm. (Air is mostly diatomic nitrogen and oxygen.)

(a) What is the final volume of this air after compression?

(b) How much work is done in compressing the air?

(c) If the temperature of the air is initially 300 K, what is the temperature after compression?

Problem 1.37. In a Diesel engine, atmospheric air is quickly compressed to about 1/20 of its original volume. Estimate the temperature of the air after compression, and explain why a Diesel engine does not require spark plugs.

Problem 1.38. Two identical bubbles of gas form at the bottom of a lake, then rise to the surface. Because the pressure is much lower at the surface than at the bottom, both bubbles expand as they rise. However, bubble A rises very quickly, so that no heat is exchanged between it and the water. Meanwhile, bubble B rises slowly (impeded by a tangle of seaweed), so that it always remains in thermal equilibrium with the water (which has the same temperature everywhere). Which of the two bubbles is larger by the time they reach the surface? Explain your reasoning fully.

Problem 1.39. By applying Newton's laws to the oscillations of a continuous medium, one can show that the speed of a sound wave is given by

$$c_s = \sqrt{\frac{B}{\rho}},$$

where ρ is the density of the medium (mass per unit volume) and B is the **bulk modulus**, a measure of the medium's stiffness. More precisely, if we imagine applying an increase in pressure ΔP to a chunk of the material, and this increase results in a (negative) change in volume ΔV, then B is defined as the change in pressure divided by the magnitude of the fractional change in volume:

$$B \equiv \frac{\Delta P}{-\Delta V/V}.$$

This definition is *still* ambiguous, however, because I haven't said whether the compression is to take place isothermally or adiabatically (or in some other way).

(a) Compute the bulk modulus of an ideal gas, in terms of its pressure P, for both isothermal and adiabatic compressions.

(b) Argue that for purposes of computing the speed of a sound wave, the *adiabatic B* is the one we should use.

(c) Derive an expression for the speed of sound in an ideal gas, in terms of its temperature and average molecular mass. Compare your result to the formula for the rms speed of the molecules in the gas. Evaluate the speed of sound numerically for air at room temperature.

(d) When Scotland's Battlefield Band played in Utah, one musician remarked that the high altitude threw their bagpipes out of tune. Would you expect altitude to affect the speed of sound (and hence the frequencies of the standing waves in the pipes)? If so, in which direction? If not, why not?

Problem 1.40. In Problem 1.16 you calculated the pressure of earth's atmosphere as a function of altitude, assuming constant temperature. Ordinarily, however, the temperature of the bottommost 10–15 km of the atmosphere (called the **troposphere**) decreases with increasing altitude, due to heating from the ground (which is warmed by sunlight). If the temperature gradient $|dT/dz|$ exceeds a certain critical value, convection will occur: Warm, low-density air will rise, while cool, high-density air sinks. The decrease of pressure with altitude causes a rising air mass to expand adiabatically and thus to cool. The condition for convection to occur is that the rising air mass must *remain* warmer than the surrounding air despite this adiabatic cooling.

(a) Show that when an ideal gas expands adiabatically, the temperature and pressure are related by the differential equation

$$\frac{dT}{dP} = \frac{2}{f+2}\frac{T}{P}.$$

(b) Assume that dT/dz is just at the critical value for convection to begin, so that the vertical forces on a convecting air mass are always approximately in balance. Use the result of Problem 1.16(b) to find a formula for dT/dz in this case. The result should be a constant, independent of temperature and pressure, which evaluates to approximately $-10°C$/km. This fundamental meteorological quantity is known as the **dry adiabatic lapse rate**.