

Second Test (2006)

This is a closed-book, closed-note test. You may use a calculator to do arithmetic, but not to store information. Please answer all questions in the space provided, using the backs of the pages if necessary.

The purpose of this test is to give you an opportunity to show me what you know and what you can do. Please present your solutions clearly and embellish them with enough English to demonstrate your mastery of the material. If you are unsure of part of a solution, don't let this stop you from completing the rest of it. Just explain your uncertainty, make the best guess you can, and proceed. In awarding partial credit I tend to respond more favorably to an honest expression of uncertainty than to a disorganized jumble of formulas.

There are 50 points on this test, and you may take up to 90 minutes to complete it. Good luck!

1. Explain and discuss each of the following terms/concepts. Use your best English. Include formulas where appropriate, but be sure to interpret them verbally.

(a) (4 points.) Chemical potential.

(b) (4 points.) Carnot cycle.

2. (8 points.) Imagine a two-state paramagnet, with 10^{23} elementary dipoles, which is *not* immersed in an external magnetic field, so that each dipole is equally likely to point up or down. Calculate the entropy of this system, in SI units. Be sure to explain your reasoning.
3. A cup of water (200 g) is “heated” in a microwave oven from 20°C to 90° .
- (a) (6 points.) By how much does the entropy of the water increase during this process? (Please express your answer in SI units.)
- (b) (4 points.) Is this entropy increase compensated for by a decrease in the entropy of some other object? Would you expect it to be? Please explain.

4. (12 points.) Imagine that the entropy of a certain object is given by the formula

$$S = bU^2,$$

where U is the object's energy and b is a positive constant whose numerical value is not important in this problem. Discuss the implications of this formula in as much detail as you can. To start with, I suggest that you calculate the temperature and then the heat capacity. Sketch a graph or two. In what way is the thermal behavior of this object unusual?

5. A *heat pump* is an electrical device that heats a building by pumping heat in from the cold outside. In other words, it's the same as a refrigerator, but its purpose is to warm the hot reservoir rather than to cool the cold reservoir (even though it does both). Let us define the following symbols, all taken to be positive by convention:

T_h = temperature inside building

T_c = temperature outside

Q_h = heat pumped into building in 1 day

Q_c = heat taken from outdoors in 1 day

W = electrical energy used by heat pump in 1 day

- (a) (2 points.) Explain why the “coefficient of performance” (COP) for a heat pump should be defined as Q_h/W .
- (b) (3 points.) What relation among Q_h , Q_c , and W is implied by energy conservation alone? Will energy conservation permit the COP to be greater than 1?
- (c) (4 points.) What relation among the symbols defined above follows from the second law of thermodynamics? (Please explain briefly.)
- (d) (3 points.) Derive a formula for the maximum possible COP, in terms of the temperatures T_h and T_c alone. (Use the back of this page if necessary.)