

Problem Set 4

(due Monday, February 10, 10:30 am)

1. **For formal written solution:** Problem 3.25 (except part f), page 108. In this problem you will see why the vibrational degrees of freedom in a solid “freeze out” at low temperature. It is this feature of heat capacities that Einstein was trying to explain when he proposed his model of a solid in 1907. As usual, please write a self-contained narrative that doesn’t refer to the parts by letter. For the graph in part (e) you could use a spreadsheet, but I recommend Mathematica. See the “Computer problems” page on the textbook’s web site for an example of how to use Mathematica’s Plot function.
2. In the previous problem set you produced a table of multiplicities for a system of two Einstein solids with $N_A = 150$, $N_B = 250$, and $q_{\text{total}} = 100$. Here is some of the data that you should have obtained:

q_A	Ω_A	q_B	Ω_B
9	1.3×10^{14}	91	2.9×10^{84}
10	2.1×10^{15}	90	7.8×10^{83}
11	3.1×10^{16}	89	2.1×10^{83}

Use this data to calculate the temperatures of both solids when $q_A = 10$. Express your answers both in fundamental units (where $k = \epsilon = 1$) and in kelvins, for $\epsilon = 0.1$ eV. Note which of the two temperatures is larger, and explain why this makes sense.

3. Problem 3.3, page 90.
4. Imagine that you spill a cup (250 g) of hot water, initially at 75°C , onto the cold ground on a winter day. The ground and everything around it are at -10°C , so the spilled water quickly cools off, gradually freezes into ice, then continues to cool. (a) Calculate the change in the entropy of the water as it cools from 75°C down to 0°C , before it freezes into ice. (b) Calculate the further change in the entropy of the water as it freezes into ice, after it has already cooled to 0°C . (c) Calculate the further change in the entropy of the now-frozen water, as it cools from 0°C down to -10°C . (The specific heat capacity of ice is very close to half that of liquid water, $2.11 \text{ J/g}\cdot^\circ\text{C}$.) (d) Calculate the change in the entropy of the surroundings, as they absorb heat from the cooling and freezing water/ice. (e) Calculate the net change in the entropy of the universe due to this process, and comment on the result.
5. Problem 3.14, page 97.
6. Problem 3.16, page 98.
7. Problem 3.20, page 107. Rest assured that, if you express each quantity as a fraction of its maximum value, the factors of N will cancel. (For the entropy, this requires some algebra.)
8. Problem 3.24, page 107. This is a spreadsheet version of Problem 1, so you can use it to (approximately) check your results in that problem. Be sure to pay attention to the detailed instructions and hints in the problem, especially the bit about the centered-difference approximation for derivatives. There is no need to re-do the estimates of ϵ for lead, aluminum, and diamond.