Physics 4610, Quantum Mechanics Name ____________________
Prof. Schroeder
Spring 2016

Problem Set 12

(due Monday, April 18, 4:00 pm)

1. This problem concerns the system of two spin-1/2 particles described in Griffiths, Subsection 4.4.3.

   (a) Apply the lowering operator to the triplet $|1\ 0\rangle$ state, and confirm that you get $\sqrt{2}\hbar|1\ {-1}\rangle$. Also show that $\sqrt{2}\hbar$ is the expected coefficient.

   (b) Apply both the raising and lowering operators to the singlet state $|0\ 0\rangle$, and confirm that in each case you get zero.

   (c) Show that the triplet states $|1\ 1\rangle$ and $|1\ {-1}\rangle$ are eigenstates of S^2, each with the appropriate eigenvalue.

2. Consider a system consisting of two subsystems, each with "one unit" of angular momentum. Each subsystem could be a particle with *orbital* angular momentum, but for definiteness, let's say they're particles with "spin 1" (that is, $s = 1$). In this problem you will "add" these two spins, working out all of the Clebsch-Gordan coefficients for this system.

   (a) List all of the basis states of the combined system for which the z component of each *individual* particle's spin is well defined. You can use any of a variety of notations for this, but be sure to define your notation clearly.

   (b) Now list all of the basis states of the combined system for which the magnitude and z component of the *total* (spin) angular momentum are well defined. Please use the "ket" notation $|s\ m\rangle$. Check that this list has the same number of entries as the list in part (a).

   (c) Two of the states in your first list are in one-to-one correspondence with two of the states in the second list. Which ones? Why?

   (d) Starting with either of the two states that you just identified, apply the (total) raising or lowering operator repeatedly to find expressions for the remaining states that have the same total s value, in terms of the part (a) basis states. (You'll need to repeatedly use the general formula for the action of a raising or lowering operator on a general eigenstate $|s\ m\rangle$.)

   (e) To work out the expressions for the remaining part (b) states, use the fact that each of these states must be orthogonal to all of the others, as well as an occasional raising or lowering operator.

   (f) Check your answers using the Clebsch-Gordan table in Griffiths.

3. Consider a Bell's theorem scenario with a system of two spin-1/2 particles, where we need to measure the spin of a particle along any of three different axes that are oriented at 120° ($2\pi/3$ radians) with respect to each other. Let us assume that these orientations all lie in the xz plane (so if the measurements are done by Stern-Gerlach

magnets, the particles are traveling in the y direction). Let θ be the angle along which the spin is measured, with the convention that $\theta = 0$ corresponds to the z axis, so in the Bell's theorem scenario we need to consider $\theta = 0$, $2\pi/3$, and $4\pi/3$. Let S_θ be the component of the spin along the θ direction.

(a) Find the 2×2 matrix of the S_θ operator, expressing each component as a function of θ. (Hint: Treat the three components of $\mathbf{\mathsf{S}}$ just like any other vector.)

(b) Check that the eigenvalues of the S_θ matrix are what you would expect, and find the corresponding normalized eigenspinors in terms of θ. Please write the eigenspinors in terms of $\cos(\theta/2)$ and $\sin(\theta/2)$.

(c) Suppose that you measure S_θ for a particle that is in the χ_+ eigenstate (that is, it has spin up along the z axis). What are the possible outcomes and their probabilities, as functions of θ?

(d) Now set $\theta = 2\pi/3$ and explain how the results are related to the Bell's theorem experiment.