Problem Set 1
(due Friday, January 15, 4:00 pm)

In all problems, be sure to make your reasoning clear and present your solutions in a way that is reasonably legible and well organized. See the syllabus for policies on obtaining assistance from others.

1. Please work the accompanying Diagnostic Test, and turn it in as soon as possible (and no later than the beginning of class on Wednesday, January 13). Although this “test” will not be graded, you will receive credit as part of this problem set for turning it in.

2. Here is some actual data that I obtained using the photoelectric effect apparatus from our introductory physics lab:

<table>
<thead>
<tr>
<th>Wavelength (nm)</th>
<th>Voltage (V)</th>
</tr>
</thead>
<tbody>
<tr>
<td>578</td>
<td>0.707</td>
</tr>
<tr>
<td>546</td>
<td>0.812</td>
</tr>
<tr>
<td>436</td>
<td>1.424</td>
</tr>
<tr>
<td>405</td>
<td>1.638</td>
</tr>
<tr>
<td>365</td>
<td>1.913</td>
</tr>
</tbody>
</table>

Use this data (or your own, if you’ve saved it from when you did that experiment) to plot a graph of voltage vs. frequency and then to obtain your best estimate of Planck’s constant. Make your reasoning clear, and express your answer both in Js and eVs.

3. Estimate (to within a factor of 2) the number of photons given off by a 10-watt LED light bulb in one second. How would your answer change for a 60-watt incandescent bulb, which looks the same to the eye?

4. The image below shows a recent electron two-slit interference experiment by Frabboni et al., Am. J. Phys. 75, 1053 (2007). At left (a) is an image of the two slits, fabricated in a 500-nm thick silicon nitride membrane using a focused ion beam. The beam of a transmission electron microscope was then aimed at the slits, and the resulting interference pattern imaged (b) with a CCD camera. Panel (c) shows the intensity of the pattern in the CCD image. Please ignore the central bright spot, which was due to the beam traveling directly through the partially transparent membrane.
(a) Measure the slit spacing and the angle between interference fringes directly from the images (using the scales shown), and use these numbers, together with what you remember about two-slit interference, to determine the wavelength of the electrons.

(b) Each electron in this beam had a kinetic energy of 200 keV. Is your measured wavelength consistent with the de Broglie relation? Explain.

5. What is the de Broglie wavelength of a fruit fly, with a mass of 0.2 mg, flying at a speed of 1 ft/s? Discuss your answer briefly.

6. A particle in a one-dimensional universe has the wavefunction $\psi(x) = Ae^{-ax^2}$, where $a = 1.2 \times 10^{18}$ in SI units.

(a) What are the units of $a$?

(b) What is the value of the normalization constant $A$, both exactly in terms of $a$ and $\pi$, and as an actual number? (Hint: To carry out the integral, change to a dimensionless variable $y$ so the exponent in the integrand is simply $-y^2$. If the integral isn’t familiar to you, do it with Mathematica.)

(c) Suppose that you now measure the position of this particle. What is the probability of finding it between $x = 0$ and $x = +0.5$ nm? (Make the same change of variables as in part (b), and do the integral numerically with Mathematica. As a reality check, make a graphical estimate of the answer.)

(d) Suppose that when you measure the position, you find the particle at $x = -0.4$ nm. Describe the wavefunction of the particle immediately after this measurement.

7. Sketch a portion (a few cycles) of the wavefunction of a particle in one dimension with a well-defined momentum that is negative. Sketch the real and imaginary parts of the wavefunction on the same graph, using different colors or dashing patterns. Then, underneath this graph, use your colored pencils to make a density-phase plot of the same wavefunction, representing the phases by colors as in the reading handout. Be sure that the various colors line up correctly with the places where the wavefunction is pure real and positive, pure imaginary and positive, pure real and negative, and so on.

8. The equation $e^{i\theta} = \cos \theta + i \sin \theta$ is the definition of $e^{i\theta}$. Use this definition and appropriate trig identities to prove that $e^{i(\theta + \phi)} = e^{i\theta}e^{i\phi}$. (This result is one example of why the definition is “sensible”; more generally, you can algebraically manipulate complex exponentials just as you would expect from the behavior of real exponentials.)

9. Use the definition of $e^{i\theta}$ to prove that $(e^{i\theta})^* = e^{-i\theta}$. Be sure to explain each step briefly.

Erwin with his psi can do
Calculations quite a few.
But one thing has not been seen:
Just what does psi really mean?
—Erich Hückel, translated by Felix Bloch