# 1. Einstein and de Broglie



Quantum mechanics began with two deceptively simple formulas:

$$E = hf \qquad \text{and} \qquad p = \frac{h}{\lambda}. \tag{1}$$

I'll refer to these as the Einstein and de Broglie relations, respectively, after the two physicists most responsible for introducing them, in the early 20th century. You're probably already familiar with both of these equations, but let's review where they came from and what they tell us.

The Einstein relation, $E = hf$, says that a particle's energy $E$ is proportional to its frequency $f$. Einstein proposed this relation for light, introducing the radical idea that light comes in discrete lumps, now called photons, each with an energy determined by the light's frequency. So a blue photon (high frequency) has more energy than a red photon (low frequency), while a gamma-ray photon has far more energy than either, and a radio-wave photon has much less. The first direct experimental evidence for the Einstein relation came from the photoelectric effect, in which high-frequency light, aimed at a metal surface, ejects electrons with more energy than low-frequency light. (Each electron, it turns out, absorbs the energy of just one photon from the light.) If you plot the energy vs. the frequency you get a straight line, whose slope is the constant of proportionality,

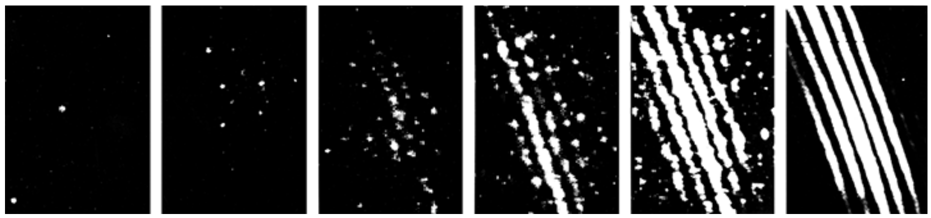
$$h = 6.63 \times 10^{-34}\ \mathrm{J\,s} = 4.14 \times 10^{-15}\ \mathrm{eV\,s}, \tag{2}$$

called *Planck's constant*.

But the Einstein relation doesn't just apply to particles of light; it applies equally well to electrons, protons, quarks, neutrinos, and (as far as we know) baseballs. Defining what we mean by the "frequency" of one of these particles is trickier than for light, so in introductory courses we usually stick to photons—and warn you not to apply $E = hf$ to anything else unless you're sure you know what you're doing. In this course you'll learn how to apply the Einstein relation to any particle at all.

The de Broglie relation, $p = h/\lambda$, says that a particle's momentum $p$ is inversely proportional to its wavelength $\lambda$. For photons, this relation is a straightforward consequence of $E = hf$ (since a light wave has $p = E/c$ and $f = c/\lambda$, where $c$ is the speed of light). But de Broglie proposed that *every* particle has a wavelength that's inversely proportional to its momentum, with the same universal constant of proportionality, $h$. The wavelength of a baseball (large $p$) is far too tiny to measure, but for low-mass particles such as electrons (small $p$), it's not hard to measure the wavelength in a diffraction experiment—as Davisson and Germer did, by accident, at about the same time as de Broglie's proposal. More recent experiments have measured the wavelengths of all sorts of subatomic particles, as well as entire atoms and molecules.

Diffraction experiments with *particles*, however, are extremely odd, because each particle can land on the detector in only one place. Here is a sequence of actual photographs from a "two-slit" interference experiment[1] performed with electrons, with the beam current increasing from left to right:



At low beam currents, you can see the distinct blips (dots) left by individual electrons on the detection screen, in apparently random locations. At higher currents, the familiar maxima and minima of the interference pattern emerge, allowing us to determine the wavelength $\lambda$ from the size of the pattern. Thus, it appears that $\lambda$ (together with the experimental geometry) determines the *probability* of an electron arriving in any particular location. Randomness and probabilities seem to be inherent in the de Broglie relation. Interference experiments with photons yield similar results: random blips at low intensity, with the wavelength-dependent pattern emerging at higher intensity.

The similarity of the Einstein and de Broglie relations becomes more apparent if we express the former in terms of the period of the wave, $T = 1/f$:

$$E = \frac{h}{T} \qquad \text{and} \qquad p = \frac{h}{\lambda}. \tag{3}$$

In other words, energy is to time (period) as momentum is to space (wavelength). Alternatively, we can write these relations in terms of the angular frequency, $\omega = 2\pi f = 2\pi/T$, and the angular wavenumber, $k = 2\pi/\lambda$. For convenience, we usually absorb the factors of $2\pi$ into Planck's *reduced* constant,

$$\hbar = \frac{h}{2\pi} = 1.05 \times 10^{-34}\ \mathrm{J\,s} = 6.58 \times 10^{-16}\ \mathrm{eV\,s} \tag{4}$$

(pronounced "h bar"), so we have simply

$$E = \hbar\omega \qquad \text{and} \qquad p = \hbar k. \tag{5}$$

Of course, the physics is exactly the same whether you use versions (1), (3), or (5).

[1] P. G. Merli, G. F. Missiroli, and G. Pozzi, "On the statistical aspect of electron interference phenomena," *American Journal of Physics* **44**, 306–307 (1976). Instead of a pair of slits, this experiment actually used a positively charged wire, perpendicular to the electron beam, allowing the electrons to pass the wire on either side and then interfere before hitting the detection screen.