20. Angular Momentum

Recall that we're looking for solutions to the TISE in spherical coordinates, for the case of a particle subject to a potential energy function V that depends only on r. We expect angular momentum to play an important role in this problem, because angular momentum would be conserved for a classical particle subject to such a potential energy function.

To treat angular momentum quantum mechanically, we'll begin by finding the appropriate operators for the three components L_x, L_y, and L_z of the angular momentum vector. We'll also work with the operator $L^2 = L_x^2 + L_y^2 + L_z^2$, corresponding to the squared magnitude of the angular momentum. Our goal is to find the eigenvalues and eigenfunctions of these operators. The good news is that we can accomplish this goal in an elegant way using ladder operators, $L_\pm = L_x \pm iL_y$, in analogy to the ladder operators for the quantum harmonic oscillator. The eigenvalues follow from the purely algebraic properties of the operators, and we can find the eigenfunctions by expressing the ladder operators in terms of derivatives and doing a bit of calculus.

Griffths covers all of this in Section 4.3, and he does a good job so I won't repeat it all here. Please study that section with great care, following every step with pencil and scratch paper, through the end of Section 4.3.1.

The bottom line is that we can find wavefunctions that are simultaneous eigenfunctions of L^2 and any *one* of the components of the $\vec{L}$ vector; conventionally we take this component to be L_z. The eigenvalues of L^2 have the form $l(l+1)\hbar^2$, where the quantum number l can be any integer. The eigenvalues of L_z are $m\hbar$, where the quantum number m ranges from $-l$ up to l in integer steps. We'll investigate the corresponding eigenfunctions in the next lesson.