

Relativity Study Guide

You should understand all the terms and definitions on the relativity handout distributed earlier, including the metric equation, $(\Delta s)^2 = (\Delta t)^2 - (\Delta x/c)^2$. From the metric equation you should be able to work essentially any quantitative problem involving the three kinds of time. You should be able to draw accurate spacetime diagrams and interpret them correctly.

You should be able to work simple problems involving length contraction. The apparent length of a moving object (as measured along the direction of motion) is *less* than its true length, by a factor of $\sqrt{1 - (v/c)^2}$.

You should understand how the invariance of the speed of light implies that simultaneous events in one reference frame are generally not simultaneous in another. You should be able to represent this fact on a spacetime diagram, showing the t and x axes for both reference frames.

You should know the Einstein velocity combination formula, $u_x = \frac{u'_x + v_x}{1 + (u'_x/c)(v_x/c)}$, where u_x and u'_x are the velocities of an object as measured in two different inertial reference frames, and v_x is the velocity of the primed frame with respect to the unprimed frame.

You should understand why momentum and energy are redefined in relativity theory (because they couldn't be conserved in all inertial reference frames otherwise). The relativistic definition of energy is $E = mc^2/\sqrt{1 - (v/c)^2}$. For an object at rest this reduces to the "rest energy", mc^2 . The difference $E - mc^2$ is called the "kinetic energy", and is approximately $\frac{1}{2}mv^2$ when $v \ll c$. The thing that's conserved for any isolated system is the *total* energy, E , summed over all objects in the system.

Nuclear Physics Study Guide

You should understand the following terminology and notation: proton, neutron, nucleon, atomic number (Z), mass number (A), isotope, atomic mass unit (u), binding energy, radioactivity, decay constant, half-life, alpha particle, beta particle, gamma ray, positron, neutrino. You should have a rough idea of the sizes and masses of nuclei and subnuclear particles, and of how much energy is typically involved in nuclear reactions. You should be able to balance nuclear reaction equations and determine what is missing in an incomplete equation. (You need not memorize the names of elements.)

Given the mass of a nucleus, you should be able to compute its binding energy. Given the masses of all the particles that participate in a nuclear reaction (such as a radioactive decay), you should be able to compute the amount of "energy released" in the reaction.

Unstable (radioactive) nuclei decay in a random fashion, with a certain fixed probability λ of decaying per unit time. For a large sample of radioactive material, this implies that the number of nuclei remaining after time t is

$$N(t) = N(0) \cdot e^{-\lambda t}.$$

You should be able to determine from this equation how λ is related to the half-life. By taking the derivative of this equation, you should also be able to derive the equation for the decay rate as a function of time.

Practice Problems

Most of these problems are taken from tests that I gave in this course in previous years. They should be good practice, and give a good picture of the level of difficulty of our upcoming final exam. Be warned, however, that they do not comprehensively cover all the material that may be on the relativity and nuclear physics portions of our final. Also please remember that the final will be comprehensive, so it will include several problems on material that was covered on our four midterm exams.

- Alice and Beverly are twins. When they are both 30 years old, Beverly leaves earth on a journey to the Sirius star system, 9 light-years away. Her spaceship travels at 90% of the speed of light. After arriving, Beverly spends one year at Sirius (studying the properties of the double star and associated planets), then returns to earth, again traveling at 90% of the speed of light.

 - Let Event D be Beverly's departure from earth, and Event S be Beverly's arrival at Sirius. Whose wristwatch measures the spacetime interval between these two events—Alice's, Beverly's, both, or neither?
 - Let Event R be Beverly's return to earth. Whose wristwatch measures the spacetime interval between Event D and Event R—Alice's, Beverly's, both, or neither?
 - How old is Alice when Beverly returns?
 - How old is Beverly when she returns?
- You are standing at the Ogden railroad station when a very long train zooms past at half the speed of light. Let Event A be the front of the train passing you, and let Event B be the rear of the train passing you. Naturally, you record the times at which these events occur (according to your wristwatch). Meanwhile, members of the train's crew (one in the engine, one in the caboose) record the times of these events as measured by their respective clocks (which they have carefully synchronized).

 - Carefully draw a spacetime diagram, showing and labeling the worldlines of you, the two ends of the train, and Events A and B.
 - Explain how the members of the train's crew can check that their clocks are synchronized, while they are in motion. (There several valid methods; please describe one of them.)
 - Whose clock(s) measure(s) the spacetime interval between Events A and B—yours or those on the train? Please explain briefly.
 - According to your wristwatch, the train takes exactly 10 seconds to pass through. From this information, knowing the speed of the train, you can calculate how long the train is. Specifically, please calculate the "true" length of the train, as it would be measured by the crew members. Express your answer in light-seconds, and don't forget to take relativistic effects into account.
 - Suppose that a child, riding on the train, carelessly drops a 1-kilogram rock out the window. To observers on the ground, this rock is hurtling forward at half the speed of light. What is the rock's kinetic energy, in joules?
- An alien spaceship whizzes past Earth at $4/5$ the speed of light, headed for Alpha Centauri. As it whizzes by, its occupants observe an explosion on the ground just below them—call this Event A. Five years later, observers on Earth detect the light from an explosion (Event B) on

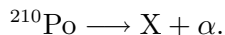
Alpha Centauri. The distance between Earth and Alpha Centauri (as measured by observers on earth) is 4 light-years.

- Carefully draw a spacetime diagram, showing and labeling Events A and B, and the worldlines of Earth, Alpha Centauri, the alien spaceship, and the light flash traveling toward Earth from Event B. Label the coordinates of Events A and B (as observed from Earth's frame) on your diagram.
 - Which event, A or B, do the aliens on the spaceship detect first? Explain briefly, with reference to your diagram.
 - Which event, A or B, *happened* first according to the aliens on the spaceship? Explain carefully, with reference to your diagram.
 - While enroute to Alpha Centauri, the alien spaceship launches two small exploratory probes—one forward toward Alpha Centauri, and the other backward toward Earth. Each probe travels at a speed of $0.9c$ with respect to the spaceship. What are the speeds and directions of the two probes with respect to Earth?
4. Tritium, or hydrogen-3 (${}^3\text{H}$), decays by beta emission with a half-life of 12.3 years. The masses of tritium and some closely related particles are as follows:

e^\pm :	0.000549 u	${}^1\text{H}$:	1.007825 u	${}^3\text{He}$:	3.016029 u
n :	1.008665 u	${}^2\text{H}$:	2.014102 u	${}^4\text{He}$:	4.002603 u
		${}^3\text{H}$:	3.016049 u		

Except for the first two, these are all *atomic* masses, that is, they include the masses of as many electrons as are normally present in the neutral atom.

- Write the reaction equation for tritium decay, specifying the identities of all particles produced.
 - Calculate the “ Q value”, or energy released, in the decay of a single tritium nucleus. Please express your answer in MeV. (Hint: Be careful to distinguish between nuclear and atomic masses.)
 - Suppose you have a sample of one gram of pure tritium. How many of the nuclei will decay in exactly one second?
5. Polonium-210 ($Z = 84$) has a half-life of 138 days, decaying by the reaction



The mass of a ${}^{210}\text{Po}$ atom is 209.982848 u, the mass of an X atom is 205.974440 u, and the mass of an alpha particle (along with enough electrons to make it a neutral atom) is 4.002603 u.

- What is X? (Please explain briefly.)
- How much energy is given off (as kinetic energy and/or gamma rays) by each polonium-210 nucleus that decays? (Please express your answer in MeV.)
- Suppose you have a sample of one gram of pure polonium-210. How many alpha particles does it emit each second?
- Combine your answers to parts (a) and (b) to calculate how many joules of energy are given off by the sample in each second. Would it warm its environment noticeably?