

Current and Magnetism Study Guide

Electric current is charge per unit time passing any given point. Negative charge counts as current going in the opposite direction.

The resistance of an object is the ratio of the voltage difference across it to the current flowing through:

$$R = \Delta V / I.$$

If this quantity is approximately constant, the object obeys “Ohm’s law”. The resistance of a wire is given by

$$R = \rho \cdot (\text{length}) / (\text{area}),$$

where ρ is the resistivity of the material.

The power converted from one form to another in any circuit element is

$$\text{power} = I \cdot \Delta V.$$

In analyzing and understanding electrical circuits, two basic principles apply:

- Junction rule: The current flowing out of any point is the same as the current flowing in.
- Loop rule: The sum of all the voltage changes around any loop of a circuit must be zero.

The voltage changes across particular circuit elements are as follows: for an ideal battery, ΔV is some fixed constant; for a conducting wire, ΔV is essentially zero; for a resistor, $\Delta V = IR$; and for a capacitor, $\Delta V = Q/C$. Depending on sign conventions for I and Q , minus signs may be needed in the last two formulas.

The cross-product of vectors \vec{A} and \vec{B} is a vector perpendicular to both \vec{A} and \vec{B} , whose direction is given by the right-hand rule and whose magnitude is $|\vec{A}||\vec{B}|\sin\theta_{AB}$.

The magnetic field \vec{B} is a bunch of little vectors living at every point in space, pointing in the same direction as the north-seeking pole of a compass needle would. The magnetic force exerted by the field is given by

$$\vec{F}_{\text{mag}} = q\vec{v} \times \vec{B} \quad \text{or} \quad \vec{F}_{\text{mag}} = (\vec{I} \times \vec{B})L,$$

where the first formula is for the force on particle with charge q and velocity \vec{v} , while the second is for the force on a segment of wire carrying current \vec{I} , with length L .

Similarly, there are two basic inverse-square formulas for the magnetic field created *by* moving charges:

$$\vec{B} = \frac{K_m q \vec{v} \times \hat{r}}{r^2} \quad \text{or} \quad \vec{B} = \sum_{\text{segments}} \frac{K_m I \vec{ds} \times \hat{r}}{r^2}.$$

In the first formula, the source is a single point charge q with velocity \vec{v} ; in the second formula, the source is a wire carrying current I , and \vec{ds} is a displacement vector along a segment of the wire. In both formulas, \hat{r} is a unit vector pointing from the source to our location, and K_m is exactly 10^{-7} in SI units. For an infinitely long, straight wire, carrying out the sum gives

$$|\vec{B}| = \frac{2K_m I}{d} \quad (\text{near a long straight wire}),$$

where d is the distance to the closest point on the wire. The direction of \vec{B} can be found using the “right-hand rule for sources.”