

Electrostatics Study Guide

You should understand the basic properties of electric charge, including the two types of charge, induced charges, charge conservation, conductors, and insulators.

Coulomb's law for the electrostatic force:

$$|\vec{F}_e| = \frac{K|q_1q_2|}{r^2}, \quad K = \frac{1}{4\pi\epsilon_0} = 8.99 \times 10^9 \frac{\text{N} \cdot \text{m}^2}{\text{C}^2}.$$

Definition of the electric field:

$$\vec{E}(x, y, z) \equiv \vec{F}_{\text{on } q_0}/q_0,$$

where q_0 is any "test charge" that you could hypothetically put at the point (x, y, z) .

Field created by a point charge:

$$|\vec{E}_{\text{of } Q}| = \frac{K|Q|}{r^2}.$$

For more complicated sources, the field is the vector sum of the individual fields ("principle of superposition"). You should be able to draw qualitatively accurate sketches of electric fields for various situations, especially those involving symmetry.

Definition of flux:

$$\Phi = |\vec{E}|A \cos \theta,$$

summed over pieces of the surface if necessary. For a closed surface, outward fluxes are positive, inward fluxes are negative. Gauss's law says that

$$\text{total flux of } \vec{E} \text{ through closed surface} = 4\pi K Q_{\text{inside}} = \frac{Q_{\text{inside}}}{\epsilon_0}.$$

Properties of conductors in equilibrium: $\vec{E} = 0$ inside; \vec{E} is perpendicular to surface outside; all excess charge is on the surface, and tends to concentrate around sharp points.

Definition of voltage (potential):

$$V(x, y, z) = U_e/q_0,$$

where q_0 is any "test charge" that you could hypothetically put at the point (x, y, z) . Like the potential energy U_e , the voltage is always relative to some arbitrary reference point ("ground"). If the source is a single point charge Q and the reference point is at infinity, then $U = KQq_0/r$. For a proton, $q_0 = 1.6 \times 10^{-19}$ C; the charge of an electron is the same size but negative. An "electron-volt" (eV) is a unit of energy equal to 1.6×10^{-19} J.

Relation between V and \vec{E} :

$$\Delta V = -\vec{E} \cdot d\vec{r}.$$

In other words, \vec{E} points from high voltage to low voltage, and the magnitude $|\vec{E}|$ is the change in voltage per unit distance as you move in that direction.

A capacitor is a device that stores positive charge Q and negative charge $-Q$, separated from each other, when a voltage difference ΔV is applied. The capacitance is defined as

$$C = Q/\Delta V,$$

charge per unit voltage. The simplest capacitor consists of two parallel plates separated by a small gap. Using the formula for the field near a plane of charge, you can show that in this case the capacitance is $\epsilon_0 A/d$, where A is the plate area and d is the separation.