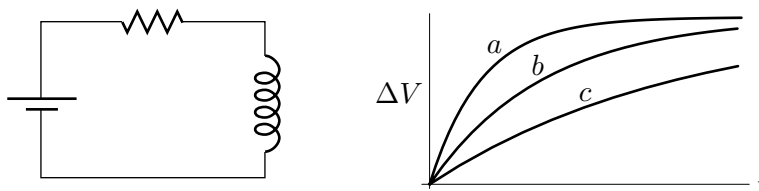
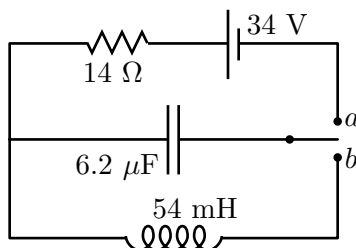


**Problem Set 8**  
(due Thursday, October 20)

- Three circuits, with identical resistors and batteries but different inductors, are wired as shown below. The graph shows the voltage difference across the resistor as a function of time for each of the three circuits. Rank the circuits according to their inductance, greatest first. Explain your reasoning carefully.

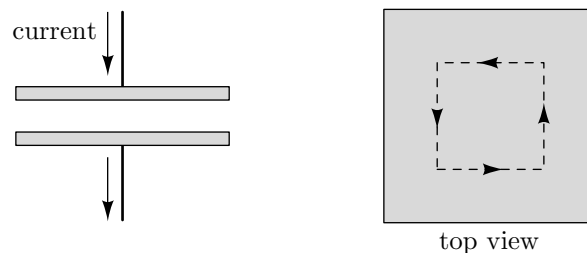


- A solenoid has length 80 cm, radius 5.0 cm, and 3000 turns distributed uniformly over its length. Its total resistance is  $10\ \Omega$ . Suppose you connect this solenoid to a 12 V battery. Exactly 5.0 milliseconds later, (a) how much energy is stored in the magnetic field in the solenoid, and (b) how much energy has been supplied by the battery up to that time?
- A solenoid is 86 cm long, has a cross-sectional area of  $17\ \text{cm}^2$ , and has 950 turns. Suppose that the current in the solenoid is 6.6 A. (a) Calculate the energy density (energy per unit volume) of the magnetic field in the solenoid. (b) Calculate the total energy stored in the magnetic field, in two ways: first, by multiplying the energy density by the total volume; and second, by calculating the inductance of the solenoid and using the formula for energy stored in an inductor.
- What must be the magnitude of a uniform electric field if it is to have the same energy density as a 0.50 T magnetic field?
- In the circuit shown below, the switch has been in position *a* for a long time. It is now thrown to position *b*. (a) Calculate the frequency of the resulting oscillating current. (b) What is the amplitude of the current oscillations?



- Explain in your own words why a capacitor acts like a large resistor at low frequency, but like a small resistor at high frequency.
- Suppose that you connect the primary coil of a transformer to an ordinary battery. Will the transformer function properly? Why or why not?

8. A transformer has 500 primary turns and 10 secondary turns. (a) If the rms voltage across the primary coil is 120 V, what is the rms voltage across the secondary coil? (b) If the secondary coil is connected to a resistive load of  $15\ \Omega$ , what are the currents in the primary and secondary?
9. An ac generator provides emf to a resistive load in a remote factory over a long, two-cable transmission line. Let the rms voltage difference between the two lines be  $V_t$ , the “transmission voltage”. Normally  $V_t$  is quite high, so a step-down transformer at the factory reduces this voltage to a lower, safer level for use in the factory. Suppose that each of the two transmission cables has a resistance of  $0.30\ \Omega$ , and the power of the generator is 250 kW. Calculate the voltage drop along the transmission lines and the rate at which energy is dissipated in these lines if (a)  $V_t = 80\ \text{kV}$ , (b)  $V_t = 8.0\ \text{kV}$ , (c)  $V_t = 800\ \text{V}$ . Comment on the acceptability of each choice.
10. Shown below is a parallel-plate capacitor with square plates, 1.0 m on a side. A downward current of 2.0 A charges the capacitor, producing a uniform (but not constant in time) electric field between the plates. (a) What is the “displacement current” through the region between the plates? (Please explain your reasoning.) (b) What is  $dE/dt$  in this region? (c) Imagine a square path, 0.5 m on a side, going around the center of the region between the plates (see top view, below right). What is the displacement current through the interior of this path? (d) What is the circulation of  $\vec{B}$  around this square path? (e) Draw a rough sketch of the magnetic field in the region between the plates.



## Study Guide

In a series  $LR$  circuit, the current turns on or off with an exponential time dependence, with a characteristic time

$$\tau = L/R.$$

In a series  $LC$  circuit, the current oscillates back and forth with angular frequency

$$\omega = 1/\sqrt{LC}.$$

The energy stored in an inductor is

$$\text{energy} = \frac{1}{2}LI^2.$$

If we think of this energy as being stored in the magnetic field, then the energy per unit volume is

$$\text{energy per unit volume} = \frac{|\vec{B}|^2}{2\mu_0}.$$

This formula is valid for any other magnetic field as well.

You should be able to predict the qualitative behavior of simple alternating-current circuits containing resistors, capacitors, and inductors. A capacitor acts like a large resistor at low frequency but a small resistor at high frequency. An inductor has the opposite effect: it acts like a large resistor at high frequency but a small resistor at low frequency.

In an ideal transformer, the “voltages” (really emf’s) on the primary and secondary coils are proportional to the number of turns:

$$\frac{V_2}{V_1} = \frac{N_2}{N_1}.$$

Since the power is the same on both sides, this implies that the currents are inversely proportional to the number of turns, or inversely proportional to the voltages.

Under non-steady-state conditions, Ampere’s law must be modified by adding the “displacement current” term:

$$(\text{circulation of } \vec{B}) = \mu_0 I + \mu_0 \epsilon_0 \frac{d\Phi_E}{dt},$$

where  $\Phi_E$  is the flux of the electric field through the same surface used for determining  $I$ . The four complete equations for the flux and circulation of  $\vec{E}$  and  $\vec{B}$  are called Maxwell’s equations; these equations can (in principle) be solved to find  $\vec{E}$  and  $\vec{B}$  for any source of charges and currents whatsoever.

The most important implication of Maxwell’s equations is that accelerating charged particles create electromagnetic waves, traveling at speed

$$c = \frac{1}{\sqrt{\epsilon_0 \mu_0}} = 3.00 \times 10^8 \text{ m/s}.$$

Within such a wave, the electric and magnetic fields are perpendicular to each other and to the direction the wave is traveling. Electromagnetic waves are created by any accelerating charged particle. The waves are strongest in the direction perpendicular to the acceleration, and carry no energy in the direction of the acceleration.