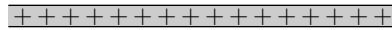
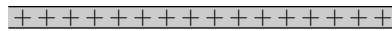
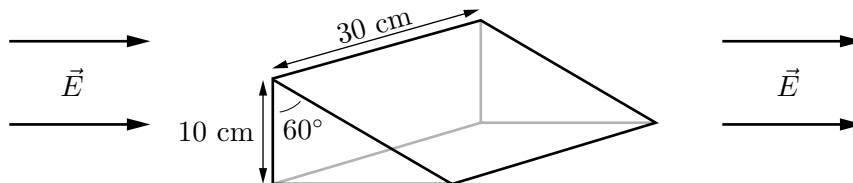


Problem Set 2
(due Friday, September 5)

- Imagine a thin rod of length L , with a total charge Q smeared uniformly along its length. A test charge is sitting at a location that's in line with the rod, a distance d beyond one of its ends. (a) Set up an integral to calculate the electric field created by the rod, at the location of the test charge. (b) Evaluate your integral to obtain a formula for the magnitude of \vec{E} ; please express your answer in terms of Q , L , d , and the constant K . (c) Check that your answer has the expected behavior in the limit $d \gg L$. (Hint: Put the complicated term in the form of a constant times $1/(1 + L/d)$, then do long division and keep only the first two terms of the series.)
- The electric field just above the surface of the charged drum of a photocopying machine has a magnitude of 2.3×10^5 N/C. What is the surface charge density on the drum, assuming that the drum is a conductor? What is the electric field *inside* the drum?
- The illustration below shows (in cross section) two parallel plates carrying identical, uniform distributions of positive charge (charge per unit area on either = σ). Assuming that the plates extend very far in all horizontal directions, what is the electric field (a) above the plates, (b) between them, and (c) below them? (Hint: use the result derived in class for the field near a single flat sheet of charge.)



- Near the earth's surface, there is typically an electric field of about 100 N/C pointing straight down. Estimate, roughly, the flux of this field through the face of a tennis racquet held horizontally. (Use any reasonable estimate for the area of the surface.) What is the flux if the racquet is turned so its face is vertical?
- The illustration below shows a wedge-shaped Gaussian surface. The electric field throughout the region is uniform, pointing directly to the right with a value of 7.8×10^4 N/C. Calculate the electric flux through (a) the vertical surface, which is perpendicular to the field; (b) the slanted surface; (c) the entire surface of the box.



- Flux Problem (see accompanying page).

7. You have four charges: $2q$, q , $-q$, and $-2q$ (where q is positive). If possible, describe how you would place a closed surface that encloses at least the charge $2q$ and through which the net electric flux is (a) 0; (b) $+3q/\epsilon_0$; and (c) $-2q/\epsilon_0$.
8. A point charge q is placed at one corner of a cube of width a . What is the flux through each of the cube faces? (Hint: Use Gauss's law and symmetry arguments.)
9. Suppose you have some closed "gaussian" surface and you know that the total flux of \vec{E} through that surface is positive. Which of the following is/are true? (a) the surface encloses a net positive charge; (b) the surface encloses only positive charges, not negative charges; (c) there is no charge outside the surface; (d) the electric field is nonzero somewhere on the surface; (e) the electric field points outward everywhere on the surface. Explain each answer briefly.
10. Suppose that instead of Coulomb's law, the equation for the magnitude of the force between two point charges were

$$|\vec{F}| = \frac{K|q_1q_2|}{r^3},$$

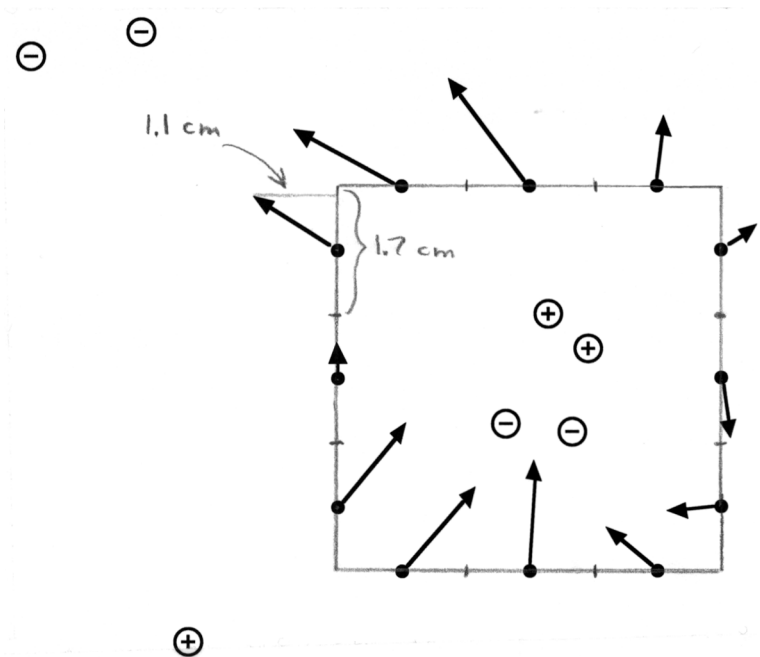
where K is some new constant with appropriate units. The electric field is defined as the force per unit charge on a point charge, as before. Consider a source consisting of a single positive point charge Q . Find a formula for the flux of the electric field through a sphere of radius R , centered on the point charge. Is Gauss's law true in this imaginary world? Why or why not?

11. Imagine a parallel-plate capacitor with the plates separated by 2 mm and a field strength inside of 50,000 N/C. The capacitor is inside a vacuum tube, so charged particles can travel between the plates without encountering air molecules. A proton is released from rest at the positive plate. What is the proton's speed when it reaches the negative plate?
12. In a typical *nuclear fission* reaction, a uranium nucleus (containing 92 protons) splits into two smaller nuclei of roughly equal size. Consider such a reaction in which the products are both palladium nuclei, each with 46 protons. Immediately after the reaction, the palladium nuclei are separated by about 10^{-14} m (the approximate size of the original uranium nucleus). At such a tiny separation, they repel each other very strongly. Taking them to be initially at rest, what is the total kinetic energy of the pair after they have separated to a great distance? The mass of a single uranium nucleus is about 4×10^{-25} kg. If an entire kilogram of uranium undergoes this fission process, roughly how many joules of energy are released? [Note: this calculation neglects the energy needed to split the uranium nucleus in the first place. However, it still gives the right order of magnitude for the energy released in fission reactions.]

Flux Problem

The figure below shows some point charges and electric field vectors that live in an imaginary two-dimensional world. (This drawing was made with an earlier version of the EField program; feel free to use the program to explore similar situations.) In this world, the electric field of a point charge is proportional to $1/r$ rather than $1/r^2$. “Surfaces” are simply one-dimensional lines, and a closed “surface” is a loop. In computing flux, the area of the surface is replaced by the length of the line segment.

The figure shows a square “surface” (really a loop), divided into twelve equal segments. The electric field at the middle of each segment is plotted, using arrows on a scale where 1 cm equals a field strength of 1 N/C. Your task is to compute the total flux of the electric field through this square loop. First compute the flux through each segment. For example, the top segment of the left side has a length of 1.7 cm, and the component of \vec{E} perpendicular to this segment is 1.1 N/C (represented by a length of 1.1 cm). The flux through this segment is therefore 1.87 N-cm/C. When you are finished, add up all the positive fluxes and negative fluxes separately, and finally calculate the total flux through the square. Given that the segments of the loop are not infinitely small, and that there is some uncertainty in how the arrows are plotted and measured, do your calculations seem to verify Gauss’s law?



Study Guide for Quiz 2

The flux of the electric field through a surface is defined as

$$\Phi = |\vec{E}|A \cos \theta,$$

where A is the area of the surface and θ is the angle between \vec{E} and an imaginary vector drawn *perpendicular* to the surface. For large surfaces, where \vec{E} and/or θ varies over the surface, you have to divide the surface into smaller pieces and compute the flux separately for each piece, then add up the little fluxes to get the total flux. For closed surfaces, outward fluxes count as positive and inward fluxes count as negative.

Gauss's law says that the total flux of \vec{E} through *any* closed surface is

$$\text{total flux through closed surface} = 4\pi K q_{\text{enclosed}} = \frac{q_{\text{enclosed}}}{\epsilon_0},$$

that is, a constant times the total net charge enclosed. This “law” is a mathematical consequence of Coulomb's law, but is sometimes more useful in understanding certain types of electric fields.

You should be able to compute changes in electrostatic potential energy for uniform fields and for collections of point charges. The energy associated with a pair of point charges, q_1 and q_2 , separated by a distance r , is

$$U_{\text{elec}} = \frac{K q_1 q_2}{r},$$

analogous to gravitational potential energy over large distances. (Note that the r is not squared.) Here the arbitrary zero-point of potential energy is taken to be when the two charges are infinitely far apart.