Physics 2220 (Schroeder) Name ______________________________
fall 2005

Problem Set 15

(due Thursday, December 8)

1. Identify the missing nucleus or particle (X) in each of the following reactions: (a) $\mathrm{X} \rightarrow {}^{65}_{28}\mathrm{Ni} + \gamma$; (b) ${}^{215}_{84}\mathrm{Po} \rightarrow \mathrm{X} + \alpha$; (c) $\mathrm{X} \rightarrow {}^{55}_{26}\mathrm{Fe} + \beta^+ + \nu$; (d) ${}^{109}_{48}\mathrm{Cd} + \mathrm{X} \rightarrow {}^{109}_{47}\mathrm{Ag} + \nu$; (e) ${}^{14}\mathrm{N} + \alpha \rightarrow {}^{17}\mathrm{O} + \mathrm{X}$.
2. A one-milligram sample of uranium metal contains 2.5×10^{18} atoms of ^{238}U. During any given second, an average of 12 of these atoms decay into ^{234}Th. From this information, find (a) the decay constant and (b) the half-life of ^{238}U.
3. Consider an initially pure 3.4-g sample of ^{67}Ga, an isotope that has a half-life of 78 hours. (a) What is its initial decay rate (in decays per second)? (b) What is its decay rate 48 hours later?
4. Uranium-238 decays into lead-206 with a half-life of 4.47×10^9 years. (Although the decay occurs in many individual steps, the first step has by far the longest half-life and therefore we can pretend that the remaining steps are "instantaneous" so the uranium converts "directly" into lead.) In a certain igneous rock, one particular crystal is found to contain 4.20 mg of ^{238}U and 2.135 mg of ^{206}Pb. Assume that any lead that was present in the original magma was incorporated into other nearby crystals, not into this one. (To check this assumption, you would want to make sure that this crystal contains no lead-204, an otherwise-common isotope that is not produced by uranium decay.) (a) How many atoms of ^{238}U and ^{206}Pb does the crystal now contain? (b) Hoe many atoms of ^{238}U did the crystal contain at formation? (c) How long ago did the rock crystallize?
5. Calculate the binding energy per nucleon of Plutonium-239.
6. You are asked to pick apart an alpha particle (^{4}He nucleus) by removing, in sequence, a proton, a neutron, and another proton. Calculate (a) the work required for each step, (b) the total binding energy of the alpha particle, and (c) the binding energy per nucleon.
7. Consider the alpha-decay of a Uranium-238 nucleus, initially at rest. Calculate the total amount of kinetic energy available to the decay products, and use momentum conservation to determine the amount of kinetic energy carried away by each. Also calculate the final velocity of each particle. (Hint: it's a good approximation to use nonrelativistic formulas.)
8. A free neutron decays into a proton, electron, and antineutrino. The kinetic energy produced in the decay can be shared by the electron and neutrino in any ratio. (The proton gets only a negligible fraction of the kinetic energy.) Look up the necessary data, and calculate the expected maximum kinetic energy of the emitted electron.
9. Heavy radionuclides emit an alpha particle rather than other combinations of nucleons because the alpha particle is such a stable, tightly bound structure. To confirm this statement, calculate the energies released (Q_3, Q_4, and Q_5 respectively) in each of the

following hypothetical decay processes and discuss the meaning of your findings.

$$\begin{array}{ll}\text{(a)} & {}^{235}\text{U} \longrightarrow {}^{232}\text{Th} + {}^{3}\text{He}; \\ \text{(b)} & {}^{235}\text{U} \longrightarrow {}^{231}\text{Th} + {}^{4}\text{He}; \\ \text{(c)} & {}^{235}\text{U} \longrightarrow {}^{230}\text{Th} + {}^{5}\text{He}.\end{array}$$

The atomic mass of ^{5}He is 5.0122 u.

10. Carbon-11 decays by β^+ (positron) emission, also emitting a neutrino in the process. The maximum observed energy of the emitted positrons is 0.960 MeV. (a) Show that the disintegration energy Q for this process is given by

$$Q = (m_{\mathrm{C}} - m_{\mathrm{B}} - 2m_e)c^2,$$

where m_{C} and m_{B} are the atomic masses of ^{11}C and ^{11}B, and m_e is the mass of a positron (or electron). (b) Calculate Q numerically, and compare to the maximum positron energy quoted above. (Hint: Remember that the *atomic* mass includes the masses of the atom's electrons. For this decay, the mass of the emitted positron is not automatically taken care of when we use atomic masses rather than nuclear masses.)

11. (a) How many atoms are contained in one kilogram of pure ^{235}U? (b) Assuming that the fissioning of each atom releases 200 MeV of energy, how much total energy, in joules, is released by the complete fissioning of one kilogram of ^{235}U? (c) For how long would this energy light a 100 watt bulb?

12. Calculate the energy released in the fission reaction $^{235}\text{U} + \text{n} \rightarrow {}^{141}\text{Cs} + {}^{93}\text{Rb} + 2\text{n}$. The atomic mass of ^{141}Cs is 140.91963 u and the atomic mass of ^{93}Rb is 92.92157 u.

Study Guide

You should understand the following terminology and notation: proton, neutron, nucleon, atomic number (Z), mass number (A), isotope, atomic mass unit (u), binding energy, radioactivity, decay constant, activity, half-life, alpha particle, beta particle, gamma ray, positron, neutrino. You should have a rough idea of the sizes and masses of nuclei and subnuclear particles, and of how much energy is typically involved in nuclear reactions.

Given the mass of a nucleus, you should be able to compute its binding energy. Given the masses of all the particles that participate in a nuclear reaction (such as a radioactive decay), you should be able to compute the amount of "energy released" in the reaction.

Unstable (radioactive) nuclei decay in a random fashion, with a certain fixed probability λ of decaying per unit time. For a large sample of radioactive material, this implies that the number of nuclei remaining after time t is

$$N(t) = N(0) \cdot e^{-\lambda t}.$$

You should be able to determine from this equation how λ is related to the half-life. By taking the derivative of this equation, you should also be able to derive the equation for the decay rate as a function of time.