Physics 2220 (Schroeder) Name ______________________
fall 2005

Problem Set 14

(due Friday, December 2)

1. By what fraction (i.e., one part in a million, or what?) is the length of an object reduced in a reference frame where it is moving 55 miles per hour? (Hint: Use the binomial approximation $(1+x)^p \approx 1+px$, which is valid whenever $|px| \ll 1$.)
2. At 11:00:00 a.m., a boiler explodes in the basement of the Museum of Modern Art in New York City (call this Event A). At 11:00:00.0003 a.m. (300 μs later), a shudder is felt at a soup factory in Camden, NJ, a distance of 150 km from Event A. (a) What is the distance from New York City to Camden in light-microseconds? (b) Draw an accurate spacetime diagram showing Events A and B. (c) An alien spaceship cruising in the direction from New York to Camden measures the Camden event to occur at the same time as the New York event. What is the approximate speed of this spaceship relative to earth? (d) Why is it impossible for Event A to have caused Event B? (Please explain from first principles, not from the impossibility of faster-than-light travel.)
3. A certain quasar is moving away from earth at $.87c$. It is emitting a jet of material toward the earth, at a speed of $.55c$ with respect to the quasar. What is the speed of this material with respect to the earth?
4. To compete with e-mail, the Post Office has recently decided to offer a new service called Super Express Mail, whereby a letter is sent to its destination at a speed of $0.999c$ using a special Letter Accelerator. For the purpose of this problem, assume that a typical letter has a mass of 25 g. (a) What is the rest energy, in joules, of a typical letter? (b) What is the total energy of the letter when it is at cruising speed? (c) What is the letter's kinetic energy at cruising speed? (d) Assuming that electrical energy costs $0.01 per million joules and that the Letter Accelerator is 100% efficient, what will be the approximate cost of a Super Express Mail postage stamp? (e) Suppose the Post Office makes a tiny mistake (unlikely as this may seem), so that the letter misses its destination and instead hits a nearby building. Describe the consequences.
5. At the Stanford Linear Acclerator, electrons are accelerated to a final energy of approximately 50 billion electron volts (50 GeV). (a) By what factor is the final energy of an electron greater than its rest energy? (b) What is the final speed of the electrons, in terms of the speed of light? (Hint: v/c is so close to 1 that it probably won't fit on your calculator. Instead, find an algebraic formula for v/c in terms of γ, and simplify it using the binomial approximation before plugging in numbers.)
6. A *positron* is a particle identical to an electron but positively charged. (Positrons are produced in certain radioactive decay processes, and also in collisions at high-energy physics laboratories.) When a positron meets an electron, the two particles can annihilate each other to produce a pair of photons. (a) Assuming that the electron and positron are initially at rest, and that the two photons come out with equal energies,

how much energy should each photon have? (b) Use conservation of momentum to explain why the two photons *must* come out with equal energies, and why there must be at least two photons (not just one).

7. Estimate (roughly) the amount by which the mass of a cup of tea increases when you warm it from room temperature to boiling temperature.
8. A *neutron star* is a collapsed star whose density is about the same as that of nuclear matter. Suppose the sun were to collapse into a neutron star. What would be its radius?
9. The radius of a certain nucleus is measured, in an electron scattering experiment, to be 3.6 fm. Roughly how many nucleons would you expect this nucleus to contain?

Study Guide

You should understand all the terms and definitions on the relativity handout distributed earlier, including the metric equation. From the metric equation you should be able to work essentially any quantitative problem involving the three kinds of time. You should be able to draw accurate spacetime diagrams and interpret them correctly.

You should be able to work simple problems involving length contraction. The apparent length of a moving object (as measured along the direction of motion) is *less* than its true length, by a factor of $\sqrt{1-(v/c)^2}$.

You should understand how the invariance of the speed of light implies that simultaneous events in one reference frame are generally not simultaneous in another. You should be able to represent this fact on a spacetime diagram, showing the t and x axes for both reference frames.

You should know the Einstein velocity combination formula,

$$u_x = \frac{u'_x + v_x}{1 + \left(\frac{u'_x}{c}\right)\left(\frac{v_x}{c}\right)},$$

where u_x and u'_x are the velocities of an object as measured in two different inertial reference frames, and v_x is the velocity of the primed frame with respect to the unprimed frame.

You should understand why momentum and energy are redefined in relativity theory (because they couldn't be conserved in all inertial reference frames otherwise). The relativistic definition of energy is

$$E = \frac{mc^2}{\sqrt{1-(v/c)^2}}.$$

For an object at rest this reduces to the "rest energy", mc^2. The difference $E - mc^2$ is called the "kinetic energy", and is approximately $\frac{1}{2}mv^2$ when $v \ll c$. The thing that's conserved for any isolated system is the *total* energy, E, summed over all objects in the system (plus any potential energy due to interactions among the objects).