Physics 2220 (Schroeder) Name ______________________________
fall 2005

Problem Set 13

(due Monday, November 28)

1. The ordinary ("Galilean") rule for combining velocities is $\vec{u} = \vec{u}' + \vec{v}$, where $\vec{u}'$ is the velocity of some object with respect to me, $\vec{u}$ is the velocity of the same object with respect to you, and $\vec{v}$ is my velocity with respect to you. (For example, if I toss a rock forward at 5 m/s while I'm moving forward at 2 m/s with respect to you, then you measure the rock to be moving at 7 m/s.) Starting with this rule for combining velocities, prove that the *acceleration* of an object is the same in any inertial reference frame. Than, assuming that forces are the same in all inertial frames, argue that if Newton's second law is true in one inertial reference frame, it is true in all inertial frames.

2. Draw an accurate spacetime diagram showing the worldlines of the following objects. Be sure to use the same size units for seconds of time and light-seconds of distance, so light signal worldlines are at 45-degree angles.

   (a) Particle A passes the point $x = 0$ at time $t = 0$ traveling at a constant speed of $(3/5)c$ in the $+x$ direction.

   (b) Particle B passes the point $x = 3$ light-seconds at $t = 0$ traveling at a constant speed of $(1/4)c$ in the $-x$ direction.

   (c) Particle C starts from rest at $x = 0$ and $t = -2$ s, and increases speed until it runs into a brick wall at $x = 5$ light-seconds and $t = 4$ s, whereupon it remains at rest thereafter.

   (d) A flash of light is emitted from position $x = 5$ ls at time $t = 1$ s and then travels in the $-x$ direction.

3. Imagine two clocks, P and Q. Both clocks leave the origin in the earth's reference frame at time $t = 0$; call this Event O. Both clocks move along the $+x$ axis, with clock P originally traveling at a speed of $(4/5)c$, while Q travels at a speed of $(1/5)c$. After a while, however, clock P slows down, comes to rest, and then begins to move back toward the origin. A short time later, it collides with the slower clock Q, which has been moving at constant speed the whole time. Call the collision Event A.

   (a) Draw a qualitatively accurate spacetime diagram of the situation just described, showing the worldlines of both clocks and Events O and A.

   (b) An observer in the earth's reference frame measures the time between events O and A with a pair of synchronized clocks S_1 and S_2 (one at the location of each event). Clocks P and Q also each measure the time between these two events. Which clock(s) measure proper time between the two events? Which clock(s) measure the spacetime interval between the two events? Which clock(s) measure coordinate time between the two events?

4. A rocket is launched from earth ($x = 0$) at time $t = 0$. The rocket travels away from earth in the $+x$ direction at a constant speed of $(3/5)c$. After 500 s as measured in the earth's frame of reference, the rocket comes to a stop. After another 200 s (in earth's frame), the rocket explodes. A burst of light from the explosion travels back to the earth. Let Event O be the departure of the rocket from earth and Event X be the explosion.

   (a) Draw an accurate spacetime diagram showing these events from the viewpoint of the earth's reference frame.

   (b) What is the coordinate time between Events O and X?

(c) What is the spacetime interval between Events O and X?

(d) What is the time between Events O and X, as measured by a clock on the rocket? (Hint: break up the journey into two pieces, before and after the rocket comes to a stop.)

5. Suppose you wish to travel to the Sirius star system, which is 9 light-years away. Being in a hurry, you'd prefer not to spend more than six years of your time on the spaceship during the one-way journey. How fast must your spaceship travel? (Please draw a spacetime diagram showing the worldlines of the earth, Sirius, and your spaceship. Use the metric equation to determine the unknown quantities.)

6. The average lifetime of a "pi meson" in its own frame of reference is 26 nanoseconds. Suppose you have a beam of pi mesons traveling at 99% of the speed of light. (a) How far would you expect them to travel before decaying (on average), if relativity were wrong and time were absolute? (b) Taking relativity into account, how far would you expect them to travel (on average) before decaying?

7. Draw an accurate spacetime diagram of a meter stick at rest (showing the worldlines of both ends), and a small bird flying past the meter stick, in the $+x$ direction, at a speed of $(2/5)c$. Let Event A be the bird passing the left end of the meter stick and let Event B be the bird passing the right end of the meter stick.

   (a) What is the time between Events A and B, as measured by observers at rest with respect to the meter stick?

   (b) What is the time between these events as measured by the bird?

   (c) According to the bird's calculations, how long is the meter stick?

8. While peacefully watching cloud formations in the desert, you suddenly see a roadrunner zip by (*beep, beep!*) at half the speed of light, pursued by a coyote running at the same speed. According to your measurements, the coyote is ten meters behind the roadrunner. How far behind does the roadrunner think the coyote is? (Hint: If the two creatures were holding a stick between them, in whose reference frame would it be moving?)

9. A 2000-kg car moving northward at 20 m/s collides and sticks to a 1500-kg car, initially at rest. (a) Use (ordinary Newtonian) momentum conservation to find the common velocity of the cars just after the collision. (b) What are the initial velocities of the two cars, as viewed from the reference frame of a third car that is traveling northward at 10 m/s? (Use the ordinary Galilean rule for combining velocities.) (c) Starting from your answer to part (a), find the common velocity of the cars, after the collision, as viewed from the third car. (d) Use the results of parts (b) and (c) to *prove* that momentum is conserved from the viewpoint of the third car.

Quid est ergo tempus? (What, then, is time?)

—Saint Augustine