

Problem Set 12

(due Thursday, November 11)

1. A quantum mechanical particle, trapped in a one-dimensional “box”, is in the $n = 5$ state (i.e., the state that is fifth-lowest in energy). (a) Sketch the wavefunction for this state. (Don’t worry about the scales of the axes.) (b) Sketch the probability density (i.e., the square of the wavefunction). (c) Looking at your second sketch, determine the most likely and least likely places of finding the particle, if you were to measure its position at this time. (d) What is the probability of finding the particle in the leftmost 10% of the box? Explain your reasoning briefly.
2. What is the ground-state energy of (a) an electron and (b) a proton if each is trapped in a one-dimensional “box” that is 0.10 nm wide (about the size of an atom)? Please express your answers in electron-volts.
3. You have an electron trapped in a one-dimensional box. Suppose that you now squeeze the box to half its former width. What happens to the energies of the states available to the electron? (Do they increase or decrease, and by what factor?)
4. An electron is trapped in a one-dimensional box that is 100 pm wide. The electron is currently in the “ground state”, or lowest-energy available state. Suppose that you now somehow measure the position of this electron (perhaps with an x-ray microscope). What is the probability of finding it somewhere within an interval 5.0 pm wide, centered at (a) $x = 25$ pm; (b) $x = 50$ pm; (c) $x = 90$ pm? (Hint: The 5-pm interval is narrow enough that you can pretend that the wavefunction is constant within it.)
5. In a *three*-dimensional, cube-shaped box, the definite-energy wavefunctions are sinusoidal functions of x , y , and z , and hence have three different “wavelengths”, λ_x , λ_y , λ_z . Each of these wavelengths is related via the de Broglie relation to the corresponding momentum component. (a) Show that the allowed energies of a particle trapped in this box are

$$E = \frac{h^2}{8mL^2}(n_x^2 + n_y^2 + n_z^2),$$

- where each of the three n ’s can be any positive integer. (b) Draw an energy-level diagram showing a dozen or so of the lowest-energy states for this particle, labeling each state with its triplet of n ’s. Note that there can be more than one state with a given energy.
6. An atom (not a hydrogen atom) absorbs a photon whose wavelength is 375 nm and then immediately emits a photon whose wavelength is 580 nm. What is the net change in the atom’s energy?
 7. A hydrogen atom is initially in its third excited state ($n = 4$). To what state should it jump to (a) emit light with the longest possible wavelength, (b) emit light with the shortest possible wavelength, (c) absorb light with the longest possible wavelength?
 8. A hydrogen atom is excited from its ground state to the $n = 4$ state. (a) How much energy is absorbed by the atom during this process? (b) Show on an energy level diagram all the possible ways (in one or more steps) that the atom can return to the ground state. Calculate the energies and wavelengths of all the possible photons that the atom could emit during this process.

9. One of the possible wavefunctions for a hydrogen atom in its first excited energy level ($n = 2$) is

$$\psi(r) = C \left(1 - \frac{r}{2a_B} \right) e^{-r/2a_B},$$

where C is a constant whose value is unimportant for our purposes. (a) Sketch (or use a computer to plot) a graph of $\psi(r)$ and a graph of $[\psi(r)]^2$. Label the horizontal axis in units of a_B , and go out to about $r = 15a_B$. Don't worry about labeling the vertical axis. (b) If the atom is in this state, where would you be *most* likely to find the electron? (c) As discussed in class, the “radial probability distribution” for a spherically symmetric wavefunction is proportional to $r^2\psi^2(r)$. Sketch a graph of *this* function. (d) From the graph you drew in part (c), estimate the value of r at which you would be most likely to find the electron. Also estimate the probability of finding the electron at $r < 2a_B$.

Study Guide

You should be able to draw the definite-energy wavefunctions for a particle in a one-dimensional “box”, and explain why only certain energies are allowed. You should be able to derive the formula for the energies,

$$E_n = \frac{h^2 n^2}{8mL^2},$$

where n is any positive integer. You should know, but not be able to derive, the comparable formula for energies of a hydrogen atom,

$$E_n = -\frac{13.6 \text{ eV}}{n^2}.$$

In general, whenever a quantum mechanical particle is confined within a limited region, its energies will be “quantized” because only certain standing-wave patterns of definite frequency will fit into the region.

You should understand the concept of transitions between energy levels, and be able to compute the photon energies and wavelengths for transitions between levels for any system whose energy levels you know.

You should be able to correctly interpret graphs of $\psi^2(r)$ for spherically symmetric wavefunctions (as in a hydrogen atom), distinguishing between the probability of finding the electron at a particular *place* and the probability of finding the electron at a particular *radius*.