Physics 2210
Schroeder

Solutions to Problem Set 9

Phonograph (a) $\alpha = \dfrac{\Delta\omega}{\Delta t} = \dfrac{\omega_f - \omega_i}{\Delta t} = \dfrac{0 - 33\frac{1}{3}\ \text{rev/min}}{.5\ \text{min}}$

$$= \underline{-66\tfrac{2}{3}\ \text{rev./min}^2}$$

if we take the original direction of rotation to be +.

In more usual units,

$$\alpha = \left(-66\tfrac{2}{3}\ \frac{\text{rev}}{\text{min}^2}\right)\left(\frac{2\pi\ \text{rad}}{1\ \text{rev}}\right)\left(\frac{1\ \text{min}}{60\ \text{s}}\right)^2 = \underline{-.116\ \text{rad/s}^2}.$$

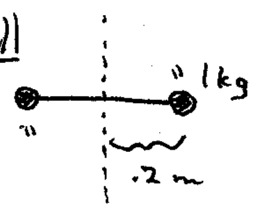
(b) In analogy to $x = x(0) + v_x(0)\cdot t + \frac{1}{2}a_x t^2$,

$$\theta = \theta(0) + \omega(0)\cdot t + \tfrac{1}{2}\alpha t^2 \quad \text{for const. } \alpha$$

$$\rightarrow \theta - \theta(0) = \left(33\tfrac{1}{3}\ \frac{\text{rev}}{\text{min}}\right)(.5\ \text{min}) - \tfrac{1}{2}\left(66\tfrac{2}{3}\ \frac{\text{rev}}{\text{min}^2}\right)(.5\ \text{min})^2$$

$$= \underline{8\tfrac{1}{3}\ \text{revolutions}}.$$

Dumbbell



(a) $I = \sum_{\text{pieces}} m_i r_i^2 = (1\text{kg})(.2\ \text{m})^2 + (1\ \text{kg})(.2\text{m})^2$

$$= \underline{.08\ \text{kg}\cdot\text{m}^2}.$$

(b) $K = \frac{1}{2} I \omega^2 = \frac{1}{2}(.08\ \text{kg}\cdot\text{m}^2)\left(3\ \frac{\text{rev}}{\text{s}}\right)^2\left(\frac{2\pi\ \text{rad}}{\text{rev}}\right)^2 = \underline{14.2\ \text{J}}.$

(c) $\text{speed} = \dfrac{2\pi r}{T} = \dfrac{2\pi\cdot(.2\ \text{m})}{1/3\ \text{s}} = \underline{3.77}\ \text{m/s}.$

(d) $K = \frac{1}{2}m_1 v^2 + \frac{1}{2}m_2 v^2 = \frac{1}{2}(1\text{kg})(3.77\ \frac{\text{m}}{\text{s}})^2 \cdot 2 = \underline{14.2\ \text{J}}.$

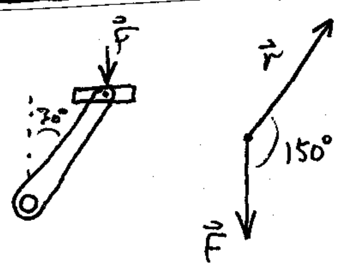
(same!)

Moments of Inertia

The greatest rotational inertia is when the mass is farthest from the rotational axis — that would be the hoop.

The least rotational inertia is when the most mass is in close to the axis: The sphere.

Bicycle Crank



(a) $\tau = |\vec{r}||\vec{F}| \sin\phi$

$= (.152\text{ m})(111\text{ N}) \sin 150°$

$= 8.44\text{ N}\cdot\text{m}$.

(b) At 90°, $\vec{F}$ and $\vec{r}$ are $\perp$, so $\tau = |\vec{r}||\vec{F}| = 16.9\text{ N}\cdot\text{m}$.

(c) At 180°, $\vec{r}$ and $\vec{F}$ are parallel, so $\tau = 0$.

Torque Practice

Torque from $\vec{F}_A$: $(8\text{ m})(10\text{ N}) \sin 135° = 56.6\text{ N}\cdot\text{m}$ (ccw)

Torque from $\vec{F}_B$: $(4\text{ m})(16\text{ N}) \sin 90° = 64.0\text{ N}\cdot\text{m}$ (clockwise)

Torque from $\vec{F}_C$: $(3\text{ m})(19\text{ N}) \sin 160° = 19.5\text{ N}\cdot\text{m}$ (ccw)

So the net torque in the c.c.w. direction is

$$\sum \tau = (56.6\text{ N}\cdot\text{m}) - (64.0\text{ N}\cdot\text{m}) + (19.5\text{ N}\cdot\text{m})$$

$$= 12.1\text{ N}\cdot\text{m}.$$

Massive Door

$$I = \sum_{\text{pieces}} m_i r_i^2 .$$ If we pretend that all pieces are at $r = 1.2$ m (half the width),

we get $I \approx M r^2 = (44000 \text{ kg})(1.2 \text{ m})^2 = 63{,}400 \text{ kg}\cdot\text{m}^2$,

which is somewhat low but not so far off.

A steady force produces a constant α, so

$$\Delta\theta = \theta - \theta_0 = \cancel{\omega(0)}^{\,0} \cdot t + \frac{1}{2}\alpha t^2 \quad \rightarrow \quad \alpha = \frac{2 \cdot \Delta\theta}{t^2} .$$

The required torque is $I\alpha$, and the required force is $\frac{\tau}{r}$

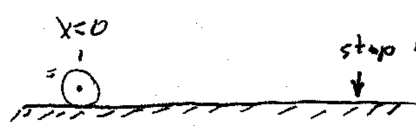
(since $\theta = 90°$). So:

$$|\vec{F}| = \frac{\tau}{r} = \frac{I\alpha}{r} = \frac{I}{r} \cdot \frac{2\Delta\theta}{t^2} = \frac{(8.7\times10^4 \text{ kg m}^2) \cdot \cancel{2}\left(\frac{\pi}{\cancel{2}}\right)}{(2.4 \text{ m})(30 \text{ s})^2}$$

$$= 126 \text{ kg}\frac{\text{m}}{\text{s}^2} = \underline{126 \text{ N}}.$$ Not so very much!

At $r = 1.2$ m, we'd need twice the force, $\underline{253 \text{ N}}$.

Rolling Wheel



(a) $$x(t) = \cancel{x(0)}^{\,0} + v_x(0) \cdot t + \frac{1}{2} a_x t^2$$

$$\cancel{v_x(t)}^{\,0} = v_x(0) + a_x t \quad \rightarrow \quad t = \frac{-v_x(0)}{a_x}$$

$$\rightarrow \quad x(t) = v_x(0)\left(\frac{-v_x(0)}{a_x}\right) + \frac{1}{2} a_x \left(\frac{v_x(0)}{a_x}\right)^2 = -\frac{(v_x(0))^2}{2a_x}$$

So $$a_x = -\frac{(v_x(0))^2}{2\,x(t)} = -\frac{(43 \text{ m/s})^2}{2 \cdot 225 \text{ m}} = \underline{-4.11 \text{ m/s}^2}.$$ (slows down)

(b) With respect to the axis, the ground and the wheel rim have a linear acceleration of 4.11 m/s^2 (never mind the direction).

So $$\alpha = \frac{a_t}{r} = \frac{4.11 \text{ m/s}^2}{.25 \text{ m}} = \underline{16.4 \text{ s}^{-2}}.$$

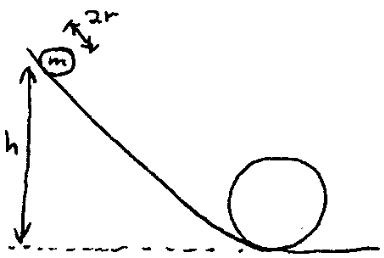
Conservation Laws Worksheet Loop-de-loop

(Adapted from Van Heuvelen, *Overview Case-Study Physics*)

1. Pictorial Representation

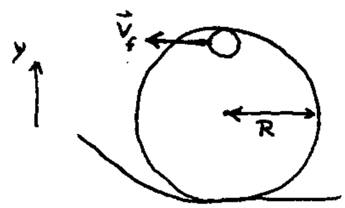
✓ symbols, including masses and velocities
✓ coordinate axes ("h")

INITIAL



FINAL

Assume $r \ll R$, so $y_f = 2R$



List "known" quantities:

R, g

ball is solid, uniform
$\rightarrow I = \frac{2}{5} m r^2$.

List unknown quantities:

h, everything else.

2. Physical Representation

Specify what the "system" is: The ball (and, technically, the earth + the track)

Which conservation law(s) apply, and why? Energy is conserved.
Gravity is a conservative force ($U = mgy$).
Normal force does no work.
No slipping $\Rightarrow$ no kinetic friction.
(Static friction causes ball to roll but does not convert energy to thermal form.)

3. Mathematical Representation

Write the appropriate conservation law(s) in mathematical form and solve.

$$E_{initial} = E_{final}$$

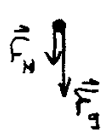
$$\Rightarrow mgh = mg(2R) + \tfrac{1}{2} m |\vec{v}_f|^2 + \tfrac{1}{2} I \omega^2$$

$$\Rightarrow h = 2R + \frac{|\vec{v}_f|^2}{2g} + \frac{1}{2mg} \cdot \frac{2}{5} m r^2 \omega^2$$

$$= 2R + \frac{|\vec{v}_f|^2}{2g} + \frac{|\vec{v}_f|^2}{5g} \quad \left(\text{no slipping} \Rightarrow |\vec{v}_f| = r\omega\right)$$

$$= 2R + \frac{7}{10} \frac{|\vec{v}_f|^2}{g}.$$

Now draw force diagram for top of loop:



$$\sum F_y = m a_y$$

$$\Rightarrow |\vec{F}_g| + |\vec{F}_N| = m \frac{|\vec{v}_f|^2}{R}.$$

Minimum $|\vec{v}_f|$ is when $|\vec{F}_N| = 0$,

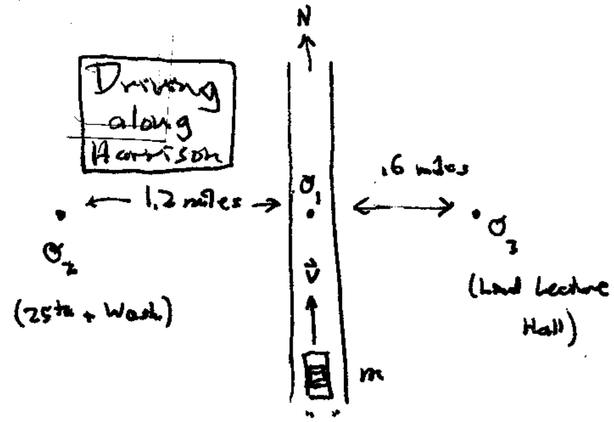
so $mg = m|\vec{v}_f|^2/R \Rightarrow |\vec{v}_f|^2 = Rg$

Thus, $h_{min} = 2R + \frac{7}{10} \frac{Rg}{g}$

$= \underline{(2.7)\,R}$.

4. Evaluation

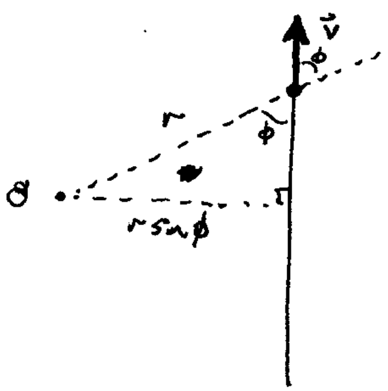
✓ correct sign?
✓ appropriate units?
✓ reasonable magnitude?



As an object moves along a straight line at constant speed, its angular momentum doesn't change. Why? Because

$$L = mr|\vec{v}|\sin\phi,$$

and $|\vec{v}|$ is constant, and $r\sin\phi$ is the perpendicular distance from the origin to the path, which is also constant.



Alternatively, an object just coasting along has no net torque on it, so its angular momentum can't possibly change.

So: for O_1, at Harrison + 36th,

$$L = m|\vec{v}|r_\perp = m|\vec{v}|\cdot 0 = \underline{0}.$$

For O_2, at 25th + Washington,

$$L = m|\vec{v}|r_\perp = (1200\text{ kg})(40\tfrac{\text{mi}}{\text{hr}})\left(\frac{1\text{ m/s}}{2.24\text{ mi/hr}}\right)(1.2\text{ mi}) \times \left(\frac{1609\text{ m}}{1\text{ mile}}\right)$$

$$= \underline{4.1\times10^{7}\ \frac{\text{kg}\cdot\text{m}^2}{\text{s}}}. \quad \text{(positive)}$$

For O_3, at Lind Lecture Hall,

$$L = -mvr_\perp = \quad -(\text{half as much})$$

$$= \underline{-2.07\times10^{7}\ \frac{\text{kg}\cdot\text{m}^2}{\text{s}}}.$$

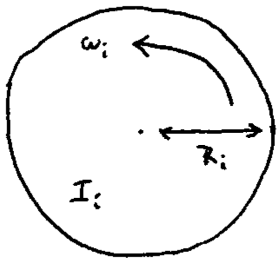
Conservation Laws Worksheet Collapsing Sun

(Adapted from Van Heuvelen, *Overview Case-Study Physics*)

1. Pictorial Representation

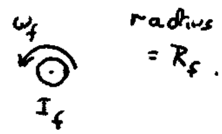
✓ symbols, including masses and velocities
___ coordinate axes (not needed)

INITIAL



M = mass

FINAL



List known quantities:

$T_i = 25$ days

$R_i \approx 100\, R_f$

List unknown quantities:

$T_f = ?$

2. Physical Representation

Specify what the "system" is: The sun.

Which conservation law(s) apply, and why? No external torques $\Rightarrow$ angular momentum is conserved.

3. Mathematical Representation

Write the appropriate conservation law(s) in mathematical form and solve.

$$L_f = L_i$$

$$\Rightarrow I_f \omega_f = I_i \omega_i$$

$$\Rightarrow I_f \cdot \frac{2\pi}{T_f} = I_i \cdot \frac{2\pi}{T_i}$$

$$\Rightarrow T_f = \frac{I_f}{I_i} T_i .$$

Let's say the sun is a uniform solid sphere both before and after.

Then $\frac{I_f}{I_i} = \frac{\frac{2}{5} M R_f^2}{\frac{2}{5} M R_i^2} = \left(\frac{R_f}{R_i}\right)^2$.

In fact the sun's density is far from uniform, so the $\frac{2}{5}$ isn't correct. So this is just a rough estimate.

Thus, $T_f = \left(\frac{R_f}{R_i}\right)^2 T_i$

$= \left(\frac{1}{100}\right)^2 (25\text{ days})$

≈ 4 minutes.

4. Evaluation

✓ correct sign?
✓ appropriate units?
✓ reasonable magnitude?

Conservation Laws Worksheet Merry-go-round

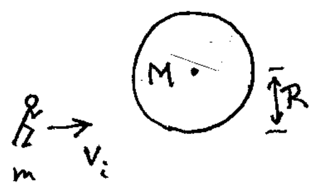
(Adapted from Van Heuvelen, *Overview Case-Study Physics*)

1. Pictorial Representation

✓ symbols, including masses and velocities

✗ coordinate axes not needed

INITIAL



FINAL

List known quantities:

$m = 44$ kg

$M = 180$ kg

$R = 1.2$ m

r = radius of gyration = .91 m

$v_i = 3$ m/s.

List unknown quantities:

I, ω

2. Physical Representation

Specify what the "system" is: child plus merry-go-round.

Which conservation law(s) apply, and why? Angular momentum: no external torques (neglecting friction)

3. Mathematical Representation Apply the appropriate conservation law(s) in mathematical form and solve.

(a) First calculate $I = Mr^2 = (180\text{ kg})(.91\text{ m})^2 = \underline{149\text{ kg}\cdot\text{m}^2}$.

(b) Child's initial $\ell = mrV\sin\phi = mRv_i = (44\text{kg})(1.2\text{m})(3\tfrac{\text{m}}{\text{s}})$

$= \underline{158.4\text{ kg}\cdot\tfrac{\text{m}^2}{\text{s}}}$.

(c) $L_{final} = L_{initial}$

$\rightarrow \ell_{\substack{child\\final}} + \ell_{\substack{m.g.r.\\final}} = L_{initial}$

$\rightarrow mRv_f + I\omega = L_{initial}$ but $\omega = \frac{v_f}{R}$

$\rightarrow mR^2\omega + I\omega = L_{initial}$

$\rightarrow \omega = \frac{L_{initial}}{mR^2 + I} = \frac{158.4\text{ kg}\cdot\frac{\text{m}^2}{\text{s}}}{(44\text{kg})(1.2\text{m})^2 + 149\text{ kg}\cdot\text{m}^2}$

$= .75\text{ s}^{-1}$ or $\underline{.75\ \frac{\text{rad}}{\text{s}}}$.

(only .12 revolutions per second)

4. Evaluation

✓ correct sign?

✓ appropriate units?

✓ reasonable magnitude?