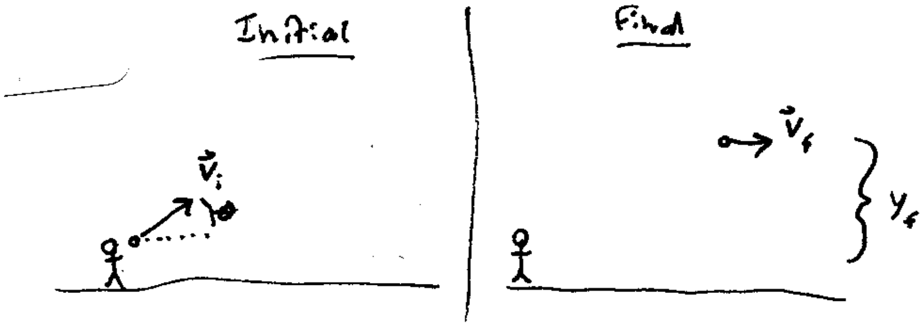
Physics 2210
Prof. Schroeder

Solutions to Problem Set 8

1. Rock Throw



$|\vec{v}_i| = 15$ m/s.
$y_i = 0$
$y_f = 10$ m

Ignoring air resistance,

$$E_{\text{final}} = E_{\text{initial}}$$

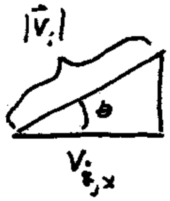
$$\rightarrow \frac{1}{2}\cancel{m}|\vec{v}_f|^2 + \cancel{m}g\,y_f = \frac{1}{2}\cancel{m}|\vec{v}_i|^2 + mg\cancel{y_i}^{\,0}$$

$$\rightarrow \frac{1}{2}|\vec{v}_f|^2 = \frac{1}{2}|\vec{v}_i|^2 - g\,y_f$$

$$\rightarrow |\vec{v}_f| = \sqrt{|\vec{v}_i|^2 - 2g\,y_f} = \sqrt{(15\tfrac{m}{s})^2 - 2(9.8\tfrac{m}{s^2})(10\,m)}$$

$$= \underline{5.4\ m/s.}$$

Horizontal component of $\vec{v}$ doesn't change, so

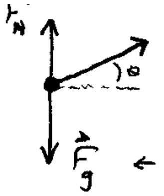


↑
(not to scale.)

$$\cos\theta = \frac{v_{i,x}}{|\vec{v}_i|} = \frac{5.4\ m/s}{15\ m/s} = .36$$

$$\rightarrow \underline{\theta = 69°} \quad \text{(above horizontal)}$$

Crate Pull



$\vec{F}_{worker} \leftarrow 210\text{ N}$

$\vec{F}_g \leftarrow (50\text{ kg})(9.8\ \tfrac{\text{N}}{\text{kg}}) = 490\text{ N}.$

$\xrightarrow{\Delta\vec{r}}$ horizontal motion

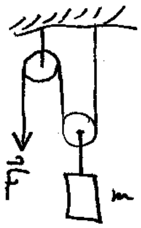
(a) $W_{worker} = |\vec{F}_{worker}||\Delta\vec{r}|\cos\theta = (210\text{ N})(3\text{ m})\cos 20° = \underline{592\text{ N·m}}$

(b) $W_{gravity} = |\vec{F}_g||\Delta\vec{r}|\cos 90° = 0.$ $\vec{F}_g$ is $\perp$ to $\Delta\vec{r}$.

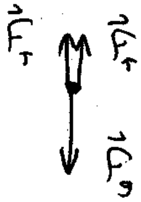
(c) $W_{normal} = |\vec{F}_N||\Delta\vec{r}|\cos 90° = 0.$ $\vec{F}_N$ is $\perp$ to $\Delta\vec{r}$.

(d) Total work $= 592\text{ N·m} + 0 + 0 = \underline{592\text{ N·m}}$.

Compound Pulley



Force diagram for hanging weight and its pulley:



$\leftarrow mg = (20\text{ kg})(9.8\ \tfrac{\text{N}}{\text{kg}}) = 196\text{ N}.$

(a) $\sum\vec{F} = 0$, so the <u>two</u> tension forces must add to 196 N, hence <u>each</u> is <u>98 N.</u>

(The upper pulley just redirects the force, so <u>I</u> pull with 98 N.)

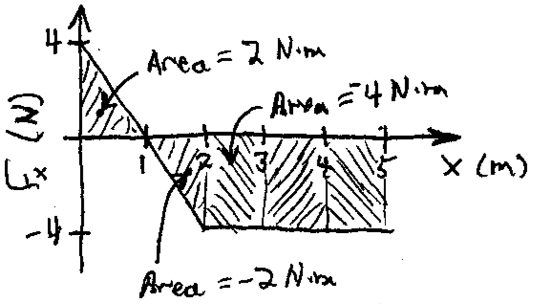
(b) To lift canister 0.02 m, I must pull cord <u>0.04</u> m.

(c) The work that <u>I</u> do pulling the cord is

$$W_{me} = (98\text{ N})(.04\text{ m}) = \underline{3.92\text{ N·m}}.$$

(d) The work done by gravity is $W_{grav.} = (196\text{ N})(-.02\text{ m}) = \underline{-3.92\text{ N·m}}$

Force Graph



Work done between:

0-1 m : 2 N·m (area of 1st Δ)

1-2 m : -2 N·m = -2 J

2-3 m : -4 J (area of □)

3-4 m : -4 J

4-5 m : -4 J

The initial Kinetic energy is $\frac{1}{2}(2\,kg)(4\,m/s)^2 = 16\,J$.

So at $x = 1\,m$, $K = 16\,J + 2\,J = 18\,J$.

at $x = 2\,m$ $K = 18\,J - 2\,J = 16\,J$

$x = 3\,m$, $K = 16\,J - 4\,J = 12\,J$

$x = 4\,m$, $K = 12\,J - 4\,J = 8\,J$

$x = 5\,m$, $K = 8\,J - 4\,J = 4\,J$.

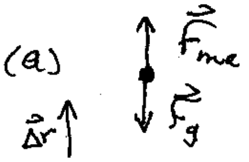
(a) At $x = 3\,m$, $K = \underline{12}\,J$.

(b) At ~~x = 4m~~, $K = 8\,J$.

(c) max K is $\underline{18\,J}$, at $x = 1\,m$.

Textbook lifting

(a)



Assuming negligible acceleration, $|\vec{F}_{me}| \approx |\vec{F}_g| = mg$,

$$\text{so } W_{me} = |\vec{F}_{me}||\vec{\Delta r}|\cos 0^\circ = mg\,(1.54\,m - .76\,m) = (2\,kg)(9.8\,\tfrac{N}{kg})(.78\,m) = \underline{15.3\,J}.$$

(b) $W_g = |\vec{F}_g||\vec{\Delta r}|\cos 180^\circ = \underline{-15.3\,J}$.

(c) $U_i = mg(.76\,m) = \underline{14.9}\,J$. $U_f = mg(1.54\,m) = \underline{30.2}\,J$.

The work that I do is the difference, $U_f - U_i = \underline{15.3\,J}$.

(d) Falling, $W_g = |\vec{F}_g||\vec{\Delta r}|\cos 0^\circ = +\underline{15.3}\,J$. There are no other forces, so the book's K.E. increases to $\underline{15.3\,J}$.

Space Shuttle

For each kilogram of shuttle,

$$K_{\text{initial}} = \frac{1}{2} m v_i^2 = \frac{1}{2}(1\text{ kg})(8000\ \tfrac{\text{m}}{\text{s}})^2 = \underline{32\text{ MJ}}.$$

$$K_{\text{final}} = \frac{1}{2} m v_f^2 = \frac{1}{2}(1\text{ kg})(100\ \tfrac{\text{m}}{\text{s}})^2 = 5000\text{ J}$$

(negligible!)

So if all the energy stays in the shuttle, each kg gains 32 MJ of thermal energy.

Convert to kilocalories:

$$(32\times10^6\text{ J})\left(\frac{1\text{ kcal}}{4186\text{ J}}\right) = \underline{7600\text{ kcal}}.$$

Melting a kg of ice requires 80 kcal.
Boiling a kg of water requires 540 kcal.
7600 kcal is huge compared to these amounts —
probably enough to vaporize any material at all.

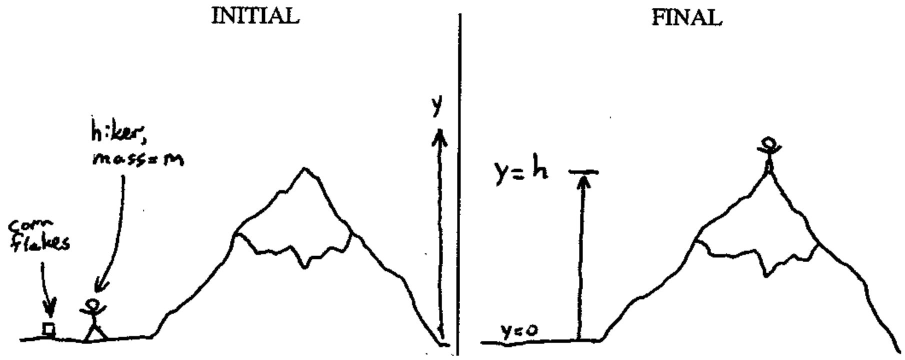
Conservation Laws Worksheet

Mt. Everest

(Adapted from Van Heuvelen, *Overview Case-Study Physics*)

1. Pictorial Representation

☐ symbols, including masses and velocities
☑ coordinate axes



List known quantities:
$h = 9000$ m
$g = 10$ m/s²
$m = 100$ kg (hiker plus gear)

List unknown quantities:
# of bowls of corn flakes.

2. Physical Representation

Specify what the "system" is: hiker + cornflakes + earth

Which conservation law(s) apply, and why? Isolated System ⇒ energy is conserved.

3. Mathematical Representation

Write the appropriate conservation law(s) in mathematical form and solve.

$$
\begin{aligned}
E_{initial} &= E_{final} \\
&= \cancelto{0}{K_{final}} + U_{final} \\
&= mgh \\
&= (100\,\text{kg})(10\,\text{m/s}^2)(9000\,\text{m}) \\
&= (9\times10^6\,\text{J})\left(\frac{1\,\text{Cal}}{4200\,\text{J}}\right) \\
&= (2100\,\text{Cal})\left(\frac{1\,\text{bcf}}{100\,\text{Cal}}\right) \\
&= 21\ \text{b.c.f.'s}.
\end{aligned}
$$

(one family-size box!)

($U_{initial} = K_{initial} = 0$, so all initial energy is in form of corn flakes.)

But even under ideal conditions, only 25% of chemical energy is converted to mechanical energy. Thus 84 bowls would be needed.

I'll round it off to 80.

4. Evaluation

☑ correct sign?
☑ appropriate units?
☑ reasonable magnitude?

Conservation Laws Worksheet

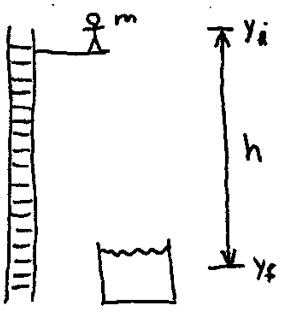
Stunt Diver

(Adapted from Van Heuvelen, *Overview Case-Study Physics*)

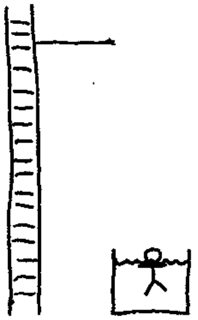
1. Pictorial Representation

✓ symbols, including masses and velocities
✓ coordinate axes

INITIAL



FINAL



List known quantities:

$h = 65\text{ m}$
$g = 10\text{ m/s}^2$
$m = 75\text{ kg}$ (I'll assume)

List unknown quantities:

ΔT

2. Physical Representation

Specify what the "system" is: Abel + water + earth

Which conservation law(s) apply, and why? The system is isolated, so the total energy, including thermal energy, is conserved.

3. Mathematical Representation

Write the appropriate conservation law(s) in mathematical form and solve.

$$E_{final} = E_{initial}$$

$$\Rightarrow E_{thermal} + mgy_f + \cancel{K_f}^{\,0} = mgy_i + \cancel{K_i}^{\,0}$$

$$\Rightarrow E_{thermal} = mg(y_i - y_f) = mgh$$

$$= (75\text{ kg})(10\text{ m/s}^2)(65\text{ m})$$

$$= (50{,}000\text{ J})\left(\frac{1\text{ Cal}}{4200\text{ J}}\right)$$

$$= 11.6\text{ Cal.}$$

The volume of the tank is

$$V = (3\text{ m})^3\left(\frac{1000\text{ liters}}{1\text{ m}^3}\right)$$

$$= 27{,}000\text{ liters.}$$

Since each Calorie can raise the temp. of 1 liter water by 1°C,

$$\underset{\text{in }^\circ\text{C}}{\Delta T} = \frac{E_{thermal}}{V} = \frac{11.6}{27{,}000}$$

$$= .0004\ ^\circ\text{C}$$

4. Evaluation

✓ correct sign?
✓ appropriate units?
✓ reasonable magnitude?
(tiny, as expected)

Niagara Falls

In one second, the gravitational potential energy lost by the water is

$$\begin{aligned} U_g &= mgy \\ &= (1.2\times10^6\ \text{kg})(9.8\ \tfrac{\text{N}}{\text{kg}})(50\ \text{m}) \\ &= 6\times10^8\ \text{J}. \end{aligned}$$

Each 60-Watt bulb consumes 60 J in one second,

so $\#\text{ of bulbs} = \dfrac{U_g}{60\ \text{J/bulb}} = \dfrac{6\times10^8\ \text{J}}{60\ \text{J/bulb}} = \underline{10^7\ \text{bulbs}}$,

This is enough to light a city!

Stair Climbing

I timed myself climbing 3 floors, or about 8 meters.

Running: 12 sec, so $\text{Power} = \dfrac{mg\,\Delta y}{\Delta t} = \dfrac{(75\ \text{kg})(10\ \frac{\text{N}}{\text{kg}})(8\ \text{m})}{12\ \text{s}}$

$= 500\ \text{Watts} = .67\ \text{hp}.$

Walking: 24 s, so only half the power:

250 Watts, or .34 hp.