Rock Throw

Initial | Find

\[ \vec{v}_i \] | \( \vec{v}_f \) \( y_f \)

Ignoring air resistance,

\[ E_{\text{initial}} = E_{\text{final}} \]

\[ \Rightarrow \frac{1}{2}m(|\vec{v}_i|)^2 + mg(y_i) = \frac{1}{2}m(|\vec{v}_f|)^2 + mg(y_f) \]

\[ \Rightarrow \frac{1}{2}(|\vec{v}_f|)^2 = \frac{1}{2}(|\vec{v}_i|)^2 - gy_i \]

\[ \Rightarrow |\vec{v}_f| = \sqrt{(|\vec{v}_i|)^2 - 2gy_i} = \sqrt{(15 \text{ m/s})^2 - 2(9.8 \text{ m/s}^2)(10 \text{ m})} \]

\[ = 5.4 \text{ m/s}. \]

Horizontal component of \( \vec{v} \) doesn't change, so

\[ \cos \theta = \frac{v_{i,x}}{v_i} = \frac{5.4 \text{ m/s}}{15 \text{ m/s}} = 0.36 \]

\[ \Rightarrow \theta = 69^\circ \quad \text{(above horizontal)} \]

(not to scale.)
Crate Roll

\[ \vec{F}_{\text{worker}} \rightarrow 210 \text{ N} \]

\[ \vec{F}_g = (50 \text{ kg})(9.8 \text{ m/s}^2) = 490 \text{ N} \]

(a) \( W_{\text{worker}} = \int F_{\text{worker}} |d\vec{r}| \cos \theta = (210 \text{ N})(3 \text{ m}) \cos 20^\circ = 592 \text{ N.m} \)

(b) \( W_{\text{gravity}} = \int F_g |d\vec{r}| \cos 90^\circ = 0 \). \( \vec{F}_g \) is \( \perp \) to \( d\vec{r} \).

(c) \( W_{\text{normal}} = \int F_N |d\vec{r}| \cos 90^\circ = 0 \). \( \vec{F}_N \) is \( \perp \) to \( d\vec{r} \).

(d) Total work = 592 N.m + 0 + 0 = 592 N.m.

Compound Pulley

\[ F = \text{Force diagram for hanging weight and its pulleys:} \]

\[ F_g \leftarrow mg = (20 \text{ kg})(9.8 \text{ m/s}^2) = 196 \text{ N} \]

(a) \( \sum F = 0 \), so the two tension forces must add to 196 N, hence each is 98 N.

(The upper pulley just redirects the force, so I pull with 98 N.)

(b) To lift container 0.002 m, I must pull cord 0.004 m.

(c) The work that I do pulling the cord is

\[ W_{\text{me}} = (98 \text{ N})(0.004 \text{ m}) = 0.392 \text{ N.m} \]

(d) The work done by gravity is

\[ W_{\text{grav}} = (196 \text{ N})(-0.02 \text{ m}) = -3.92 \text{ N.m} \]
Force Graph

Work done between:
0-1 m: 2 N·m (area of 1st Δ)
1-2 m: -2 N·m = -2 J
2-3 m: -4 J (area of □)
3-4 m: -4 J
4-5 m: -4 J

The initial kinetic energy is \( \frac{1}{2} (2 \text{ kg}) (4 \text{ m/s})^2 = 16 \text{ J} \).

So at \( x = 1 \text{ m}, \ K = 16 \text{ J} + 2 \text{ J} = 18 \text{ J} \),
\[ \begin{align*}
& a) \text{ at } x = 3 \text{ m}, \quad K = 12 \text{ J} , \\
& b) \text{ at } x = 4 \text{ m}, \quad K = 8 \text{ J} , \\
& c) \text{ max } K \text{ at } 18 \text{ J}, \\
& a) \text{ at } x = 1 \text{ m} .
\end{align*} \]

Textbook lifting

(a) \uparrow F_{\text{me}} \quad \text{Assuming negligible acceleration, } |F_{\text{me}}| = |F_g|
\[ F_{\text{me}} \uparrow \downarrow F_g \]

so \( W_{\text{me}} = |F_{\text{me}}| \cdot 1.54 \text{ m} \cdot \cos 0^\circ = mg \)
\[ = mg (1.54 \text{ m} \cdot 1.76 \text{ m}) = (2 \text{ kg}) (9.8 \text{ N/kg}) (1.78 \text{ m}) = 15.3 \text{ J} \]

(b) \( W_g = |F_g| \cdot 1.54 \text{ m} \cdot \cos 180^\circ = -15.3 \text{ J} \)

(c) \( U_i = mg (1.76 \text{ m}) = 14.9 \text{ J}, \quad U_f = mg (1.54 \text{ m}) = 30.2 \text{ J} \)

The work that I do is the difference, \( U_f - U_i = 15.3 \text{ J} \).

(d) Falling, \( W_g = 15.3 \text{ J} \cdot \cos 0^\circ = +15.3 \text{ J} \).

There are no other forces, so the book's K.E. increases to 15.3 J.
Space Shuttle

For each kilogram of shuttle,
\[ K_{\text{initial}} = \frac{1}{2} m v_i^2 = \frac{1}{2} (1 \text{ kg})(8000 \text{ m/s})^2 = 32 \text{ MJ}. \]
\[ K_{\text{final}} = \frac{1}{2} m v_f^2 = \frac{1}{2} (1 \text{ kg})(100 \text{ m/s})^2 = 5000 \text{ J} \] (negligible)

So if all the energy stays in the shuttle each kg gains 32 MJ of thermal energy.

Convert to kilocalories:
\[ (32 \times 10^6 \text{ J})(\frac{1 \text{ kcal}}{4186 \text{ J}}) = 7600 \text{ kcal}. \]

Melting a kg of ice requires 80 kcal.
Boiling a kg of water requires 540 kcal.

7600 kcal is huge compared to these amounts — probably enough to vaporize any material at all!
Conservation Laws Worksheet

1. Pictorial Representation

   INITIAL
   - symbols, including masses and velocities
   - coordinate axes

   FINAL
   - $y = h$
   - $y = 0$

   List known quantities:
   - $h = 9000 \text{ m}$
   - $g = 10 \text{ m/s}^2$
   - $m = 100 \text{ kg}$ (hiker + gear)

   List unknown quantities:
   - # of bowls of corn flakes.

2. Physical Representation

   Specify what the "system" is:
   - hiker + corn flakes + earth

   Which conservation law(s) apply, and why?
   - Isolated system $\Rightarrow$ energy is conserved.

3. Mathematical Representation

   Write the appropriate conservation law(s) in mathematical form and solve.

   \[
   E_{\text{initial}} = E_{\text{final}}
   \]

   \[
   = K_{\text{final}} + U_{\text{final}}
   \]

   \[
   = mgh
   \]

   \[
   = (100 \text{ kg})(10.75 \text{ m})(9000 \text{ m})
   \]

   \[
   = (7 \times 10^6 \text{ J})(\frac{1 \text{ Cal}}{4200 \text{ J}})
   \]

   \[
   = (2100 \text{ Cal})(\frac{1 \text{ bcf}}{100 \text{ Cal}})
   \]

   \[
   = 21 \text{ bcf}.
   \]

   (one family-size box)

   \[
   (U_{\text{initial}} = K_{\text{initial}} = 0)
   \]

   So all initial energy is
   - in form of corn flakes.

   But even under ideal conditions, only 25% of chemical energy is converted to mechanical energy. Thus 84 bowls would be needed.

   I'll round it off to 80.

4. Evaluation

   - correct sign?
   - appropriate units?
   - reasonable magnitude?
1. Pictorial Representation

- Symbols, including masses and velocities
- Coordinate axes

INITIAL

FINAL

List known quantities:
- $h = 65 \text{ m}$
- $g = 10 \text{ m/s}^2$
- $m = 7.5 \text{ kg}$ (I'll assume)

List unknown quantities:
- $\Delta T$

2. Physical Representation

Specify what the "system" is: $\text{ Abel } + \text{ water } + \text{ earth}$

Which conservation law(s) apply, and why? The system is isolated, so the total energy, including thermal energy, is conserved.

3. Mathematical Representation

Write the appropriate conservation law(s) in mathematical form and solve.

$$E_{\text{final}} = E_{\text{initial}}$$

$$\Rightarrow E_{\text{thermal}} + mgy_f + \frac{1}{2}v_f^2 = mgy_i + \frac{1}{2}v_i^2$$

$$\Rightarrow E_{\text{thermal}} = m(g(y_i - y_f)) = mgh$$

$$= (75 \text{ kg})(10 \text{ m/s}^2)(65 \text{ m})$$

$$= (50,000 \text{ J})(\frac{1 \text{ Cal.}}{4200 \text{ J}})$$

$$= 11.6 \text{ Cal.}$$

The volume of the tank is

$$V = (3 \text{ m}^3)(\frac{1000 \text{ liters}}{1 \text{ m}^3})$$

$$= 27,000 \text{ liters}.$$  

Since each Calorie can raise the temp. of 1 liter water by 1°C,

$$\Delta T = \frac{E_{\text{thermal}}}{V} = \frac{11.6}{27,000 \text{ °C}}$$

$$= 0.0004 \text{ °C},$$

4. Evaluation

- Correct sign?
- Appropriate units?
- Reasonable magnitude?
  (tiny, as expected)
Niagara Falls

In one second, the gravitational potential energy lost by the water is

\[ U_g = m g y \]
\[ = (1.2 \times 10^6 \text{ kg})(9.8 \frac{\text{m}}{\text{kg}})(50 \text{ m}) \]
\[ = 6 \times 10^9 \text{ J}. \]

Each 60-Watt bulb consumes 60 J in one second, so

\[ \text{# of bulbs} = \frac{U_g}{60 \text{ J/bulb}} = \frac{6 \times 10^9 \text{ J}}{60 \text{ J/bulb}} = 10^7 \text{ bulbs}. \]

This is enough to light a city!

Stair Climbing

I timed myself climbing 3 flights or about 8 meters.

**Running:** 12 sec, so Power = \( \frac{m g d y}{t} = \frac{(75 \text{ kg})(10 \frac{\text{m}}{\text{s}})(8 \text{ m})}{12 \text{ s}} \]
\[ = 500 \text{ Watts} = .67 \text{ hp}. \]

**Walking:** 26 sec, so only half the power:

250 watts or .34 hp.