

Solutions to Problem Set 7

1.  $K = \frac{1}{2} m \bar{v}^2$ , so if the speed of an object is doubled, its kinetic energy quadruples. Proof: If  $v_f = 2v_i$ , then  $K_f = \frac{1}{2} m v_f^2 = \frac{1}{2} m (2v_i)^2 = 4 \cdot \frac{1}{2} m v_i^2 = 4 \cdot K_i$ .

2. (a) Linebacker:  $K = \frac{1}{2} (110 \text{ kg}) (8.1 \text{ m/s})^2 = \underline{3610 \text{ J}}$ .

(b) Bullet:  $K = \frac{1}{2} (.0042 \text{ kg}) (950 \text{ m/s})^2 = \underline{1900 \text{ J}}$ .

(c) Nimitz:  $K = \frac{1}{2} (9,400 \text{ tons}) (32 \text{ knots})^2 \left( \frac{907 \text{ kg}}{1 \text{ ton}} \right) \left( \frac{1.69 \text{ ft/s}}{1 \text{ knot}} \right)^2 \times \left( \frac{.305 \text{ m}}{1 \text{ ft.}} \right) = \underline{1.13 \times 10^{10} \text{ J}}$ .

3. For each plumb, neglecting air resistance,

$$E_{\text{final}} = E_{\text{initial}} \quad (\text{"final" = 5 m altitude})$$

$$\rightarrow K_f + U_{g,f} = K_i + U_{g,i}$$

$$\rightarrow K_f = K_i + U_{g,i} - U_{g,f} \quad \leftarrow \begin{array}{l} \text{Same for all 3} \\ \text{plumbs (same} \\ \text{"final" altitude)} \end{array}$$

$\uparrow$  Same for all 3 plumbs (same starting height)  
 $\uparrow$  Same for all 3 plumbs (same speed)

So  $K_f$  must also be the same for all 3, and therefore they all have the same speed.

But: They're moving in different directions, so they don't have the same velocities.

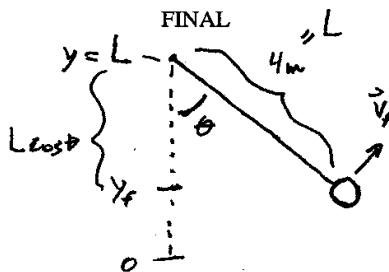
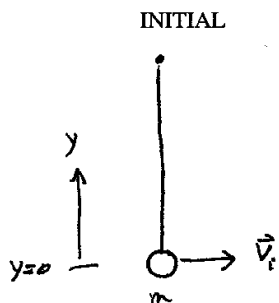
# Conservation Laws Worksheet

## Pendulum

(Adapted from Van Heuvelen, Overview Case-Study Physics)

### 1. Pictorial Representation

✓ symbols, including masses and velocities  
 ✓ coordinate axes



List known quantities:

$$L = 4 \text{ m}$$

$$m = 2 \text{ kg}$$

$$|\vec{v}_i| = 8 \text{ m/s}$$

$$\theta = 60^\circ$$

List unknown quantities:

$$|\vec{v}_f| = ?$$

$$\text{max angle} = \theta_{\text{max}} = ?$$

$$E = ?$$

### 2. Physical Representation

Specify what the "system" is: Pendulum (interacting with earth)

Which conservation law(s) apply, and why? Energy is conserved, if we neglect air resistance. Gravity force has a potential energy function, while the tension force does no work. ( $\perp$  to  $\vec{v}$ ).

### 3. Mathematical Representation

Apply the appropriate conservation law(s) in mathematical form and solve.

$$(a) \quad E_f = E_i$$

$$\rightarrow K_f + U_f = K_i + U_i$$

$$\rightarrow \frac{1}{2} m v_f^2 + m g y_f = \frac{1}{2} m v_i^2 + m g y_i$$

$$\rightarrow \frac{1}{2} v_f^2 = \frac{1}{2} v_i^2 + g(y_i - y_f)$$

$$\rightarrow v_f^2 = v_i^2 - 2g y_f$$

$$= v_i^2 - 2g(L - L \cos \theta)$$

$$= v_i^2 - 2gL(1 - \cos \theta)$$

$$= (8 \frac{\text{m}}{\text{s}})^2 - 2(9.8 \frac{\text{m}}{\text{s}^2})(4 \text{ m})(1 - \cos 60^\circ)$$

$$= 24.8 \frac{\text{m}^2}{\text{s}^2}$$

$$\rightarrow v_f = \sqrt{24.8 \frac{\text{m}^2}{\text{s}^2}} = 5.0 \text{ m/s}$$

$$(b) \quad \text{At } \theta_{\text{max}}, \quad K_f = 0 \text{ so}$$

$$U_f = K_i \rightarrow m g y_f = \frac{1}{2} m v_i^2$$

$$\rightarrow y_f = \frac{v_i^2}{2g} = \frac{(8 \text{ m/s})^2}{2(9.8 \text{ m/s}^2)} = 3.27 \text{ m}$$

or .73 m below pivot.

$$\text{So } L \cos \theta_{\text{max}} = .73 \text{ m}$$

$$\rightarrow \theta_{\text{max}} = \cos^{-1} \left( \frac{.73 \text{ m}}{4 \text{ m}} \right) = 79^\circ$$

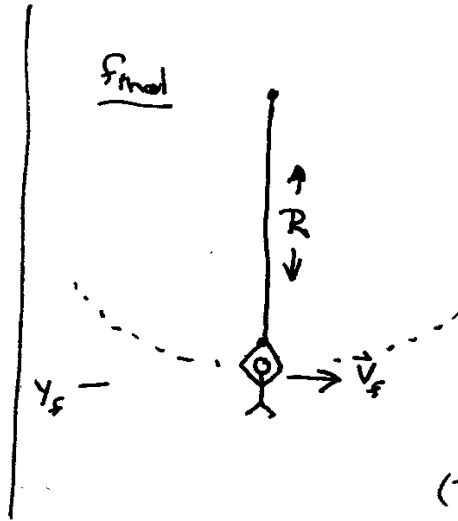
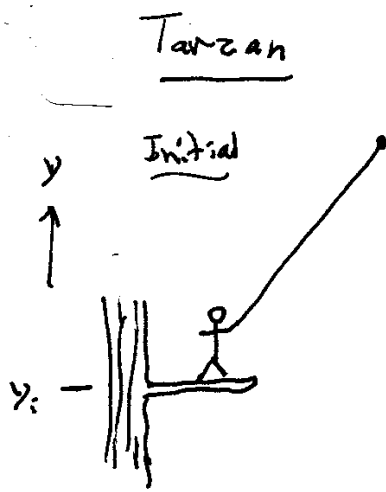
$$(c) \quad \text{I'll compute } E \text{ at the bottom: } E = K + U = K + 0$$

$$= \frac{1}{2} m |\vec{v}_i|^2 = \frac{1}{2} (2 \text{ kg})(8 \text{ m/s})^2$$

$$= 64 \text{ J.}$$

### 4. Evaluation

- ✓ correct sign?
- ✓ appropriate units?
- ✓ reasonable magnitude?



$$y_i = 5.0 \text{ m}$$

$$y_f = 1.0 \text{ m}$$

$$m = 80 \text{ kg}$$

$$R = 10 \text{ m}$$

$$|\vec{F}_T| = ?$$

(Tension is greatest at bottom of swing, where speed is greatest.)

Conservation of energy applies, if we neglect air resistance. (Vines does no work.)

$$E_f = E_i$$

$$\rightarrow K_f + U_f = K_i + U_i$$

$$\rightarrow \frac{1}{2} m |\vec{v}_f|^2 + m g y_f = m g y_i \rightarrow |\vec{v}_f|^2 = 2g |y_f - y_i|$$

Force diagram at bottom:



$$\sum F_y = m a_y \rightarrow |\vec{F}_T| - |\vec{F}_g| = m \frac{|\vec{v}_f|^2}{R}$$

$$\rightarrow |\vec{F}_T| = m \frac{|\vec{v}_f|^2}{R} + m g$$

$$= (80 \text{ kg}) \left( \frac{2g(y_f - y_i)}{R} + g \right)$$

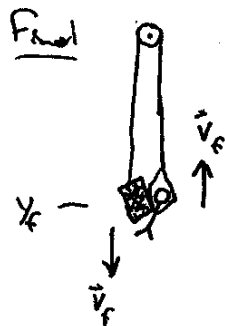
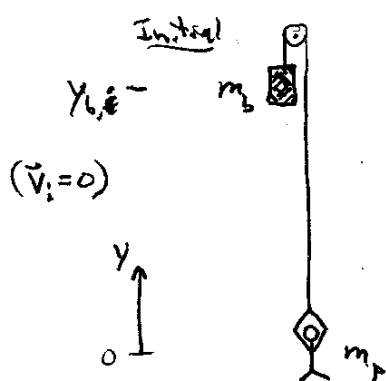
$$= (80 \text{ kg}) \left( \frac{2(9.8 \frac{\text{N}}{\text{kg}})(4 \text{ m})}{10 \text{ m}} + 9.8 \frac{\text{N}}{\text{kg}} \right)$$

$$= (80 \text{ kg}) (9.8 \frac{\text{N}}{\text{kg}}) (.8 + 1)$$

$$= \underline{1410 \text{ N}}$$

Solutions to Problem Set 7

Paddy + the Barrel (#2)



- $y_{b,i} = 50 \text{ m}$
- $y_f = 25 \text{ m}$
- $m_p = 75 \text{ kg}$
- $m_b = 100 \text{ kg}$
- $|\vec{v}_i| = 0$
- $|\vec{v}_f| = ?$  (same speed for both)

The "system" is Paddy + bricks + earth.

Energy conservation applies if there is no friction.

(The tension force of the rope does no net work on the system, since it does positive work on P. but negative work on the barrel and these cancel.)

$$E_f = E_i$$

$$\rightarrow K_{p,f} + K_{b,f} + U_{p,f} + U_{b,f} = K_{p,i} + K_{b,i} + U_{p,i} + U_{b,i}$$

$$\rightarrow \frac{1}{2} m_p |\vec{v}_f|^2 + \frac{1}{2} m_b |\vec{v}_f|^2 + m_p g y_f + m_b g y_f = m_b g y_{b,i}$$

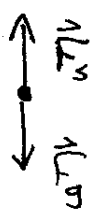
$$\rightarrow \frac{1}{2} (m_p + m_b) |\vec{v}_f|^2 + (m_p + m_b) g y_f = m_b g y_{b,i} = m_b g \cdot 2y_f$$

$$\rightarrow \frac{1}{2} (m_p + m_b) |\vec{v}_f|^2 = g y_f [2m_b - (m_p + m_b)] = (m_b - m_p) g y_f$$

$$\rightarrow |\vec{v}_f|^2 = 2g y_f \cdot \frac{m_b - m_p}{m_b + m_p} = 2(9.8 \frac{\text{m}}{\text{s}^2})(25 \text{ m}) \cdot \left( \frac{100 \text{ kg} - 75 \text{ kg}}{100 \text{ kg} + 75 \text{ kg}} \right)$$

$$= 490 \frac{\text{m}^2}{\text{s}^2} \cdot \frac{25}{175} = 70 \frac{\text{m}^2}{\text{s}^2} \rightarrow |\vec{v}_f| = \underline{\underline{8.4 \text{ m/s}}}$$

## Rubber Band



$$\begin{aligned}\Sigma F_y &= ma_y \\ \rightarrow |\vec{F}_s| - |\vec{F}_g| &= 0 \\ \rightarrow |\vec{F}_s| &= mg.\end{aligned}$$

But  $|\vec{F}_s| = k_s$  (amount of stretch)

$$\begin{aligned}\text{So } k_s &= \frac{mg}{\text{stretch}} \\ &= \frac{(1.5 \text{ kg})(9.8 \text{ N/kg})}{.1 \text{ m}} \\ &= \underline{49 \text{ N/m}}.\end{aligned}$$

If the force doubles, so does the amount of stretch, so it'll stretch 20 cm to hold up a total mass of 1 kg.

## Catapult

(a)  $|\vec{F}_s| = k_s \cdot x = (100 \text{ N/m})(5 \text{ m}) = \underline{500 \text{ N}}$ .

(b)  $U_s = \frac{1}{2} k_s x^2 = \frac{1}{2} (100 \text{ N/m})(5 \text{ m})^2 = \underline{1250 \text{ J}}$ .

(c)  $E_f = E_i$

$$\rightarrow K_f + U_f = K_i + U_i$$

$$\rightarrow K_f + 0 = 0 + U_i$$

$$\rightarrow \frac{1}{2} m v_f^2 = U_i$$

$$\rightarrow v_f^2 = \frac{2U_i}{m}$$

$$\rightarrow v_f = \sqrt{\frac{2U_i}{m}}$$

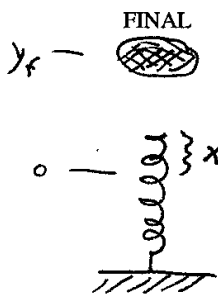
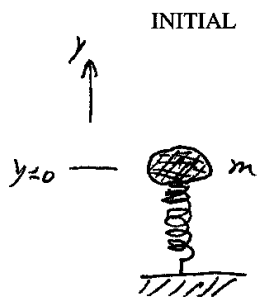
$$= \sqrt{\frac{2 \cdot 1250 \text{ J}}{1 \text{ kg}}}$$

$$= \underline{50 \frac{\text{m}}{\text{s}}} \quad (!)$$

(That's 115 mph!)

1. Pictorial Representation

- symbols, including masses and velocities
- coordinate axes



List known quantities:

$$\vec{v}_i = \vec{v}_f = 0$$

$x = \text{amount of compression} = 40 \text{ cm.}$

$$m = 8 \text{ kg}$$

List unknown quantities:

$$y_f = ?$$

2. Physical Representation

Specify what the "system" is: Stone, interacting with the spring and the earth.

Which conservation law(s) apply, and why? Energy is conserved, since both spring force and gravity ~~are~~ have potential energy functions.  
(Neglect air resistance)

3. Mathematical Representation

Apply the appropriate conservation law(s) in mathematical form and solve.

(a) The spring force balances the stone's weight, so  $k = \frac{|F_s|}{x} = \frac{mg}{x} = \frac{(8 \text{ kg})(9.8 \frac{\text{N}}{\text{kg}})}{.1 \text{ m}} = 784 \frac{\text{N}}{\text{m}}$ .

(b)  $U_{\text{spring},i} = \frac{1}{2} k x^2 = \frac{1}{2} (784 \frac{\text{N}}{\text{kg}}) (.1 \text{ m})^2 = 62.7 \text{ J}$ .

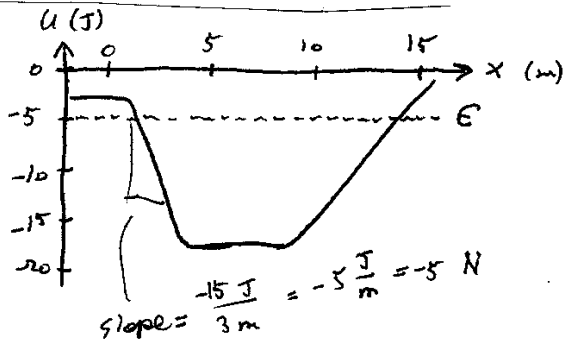
(c)  $E_{\text{final}} = E_{\text{initial}} \rightarrow K_f + U_{g,f} + U_{s,f} = K_i + U_{g,i} + U_{s,i}$   
 $\rightarrow U_{g,f} - U_{g,i} = U_{s,i} = 62.7 \text{ J}$ ,  $\leftarrow U_g$  increases by this much.

(d)  $mg y_f - mg y_i = 62.7 \text{ J}$   
 $\rightarrow y_f - y_i = \frac{62.7 \text{ J}}{(8 \text{ kg})(9.8 \frac{\text{N}}{\text{kg}})} = .80 \text{ m}$ .

4. Evaluation

- correct sign?
- appropriate units?
- reasonable magnitude?

## Potential Energy Graph



(a)  $F_x = -(\text{slope}) = +5 \text{ N}$  at  $x=2 \text{ m}$ .

(b) At  $x=2 \text{ m}$ ,  $K = \frac{1}{2} m v^2 = \frac{1}{2} (2 \text{ kg}) (1.5 \frac{\text{m}}{\text{s}})^2 = 2.25 \text{ J}$ .

while  $U \approx -7 \text{ J}$  (approximately)

so  $E = K + U \approx -5 \text{ J}$ .

So limits of motion are between about  $x=1.5 \text{ m}$  and  $x=14 \text{ m}$ , where  $U=E$  so  $K=0$ .

(c) At  $x=7 \text{ m}$ ,  $U = -17 \text{ J}$  so

$K = E - U = (-5 \text{ J}) - (-17 \text{ J}) = 12 \text{ J}$ .

$\rightarrow |v| = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2(12 \text{ J})}{2 \text{ kg}}} = \underline{3.5 \text{ m/s}}$ .