Paddy and the Barrel

(a) \[ \sum F_x = m_1 a_x \rightarrow F_T - m_1 g = m_1 a_x \]

Barrel: \[ \sum F_x = m_2 a_x \rightarrow m_2 g - F_T = m_2 a_x \]

Eliminate \( F_T \): \[ m_1 (a_x + g) = m_2 (g - a_x) \]

Solve for \( a_x \): \[ (m_1 + m_2) a_x = (m_2 - m_1) g \]
\[ a_x = \left( \frac{m_2 - m_1}{m_2 + m_1} \right) g = \frac{25 \text{ kg}}{175 \text{ kg}} \cdot g = 1.4 \text{ m/s}^2 \]

(b) For constant \( a_x \),
\[ v_x(t) = v_x(0) + a_x \cdot t \]
\[ x(t) = x(0) + v_x(0) \cdot t + \frac{1}{2} a_x \cdot t^2 \rightarrow t = \sqrt{\frac{2 \cdot x(t)}{a_x}} \]
So \[ v_x(t) = a_x \cdot \sqrt{\frac{2 \cdot x(t)}{a_x}} = \sqrt{2 \cdot a_x \cdot x(t)} \]
\[ = \sqrt{2 \cdot (1.4 \text{ m/s})(25 \text{ m})} = 8.4 \text{ m/s} \]

(Of course, the barrel is going equally fast coming down...)
Constrained Motion Problem Worksheet

1. Pictorial Representation
   - Coordinate axes
   - Sketch of situation, labeled with symbols

   List known quantities:
   - \( m_1 = 1000 \text{ kg} \)
   - \( m_2 = 500 \text{ kg} \)
   - \( t_{\text{max}} \theta = 1 \)
   - \( \mu_s = 0.18 \)

   List unknown quantities:
   - Is this possible?
   - Calculate maximum angle, \( \theta \), and compare

2. Physical Representation
   - Describe the constraint: Straight-line motion, \( \ddot{a} = 0 \).
   - Force diagram for each object, listing agent for each force
   - Net force points in direction of acceleration

   Car:
   - \( F_c \)
   - \( F_{\text{net}} \)

   Trailer:
   - \( F_t \)
   - \( F_{\text{net}} \)

   *I'll assume no friction on the trailer.*

3. Mathematical Representation
   - Apply Newton's second law in component form and solve.
   - Do not plug in numerical values until the very end.

   For the trailer:
   \[
   \sum F_x = m_2 a_x \Rightarrow |\vec{F}_x| - |\vec{F}_y| \sin \theta = 0 \Rightarrow |\vec{F}_x| = m_2 g \sin \theta.
   \]

   For the car:
   \[
   \sum F_y = m_1 a_y \Rightarrow |\vec{F}_y| - |\vec{F}_z| \cos \theta = 0 \Rightarrow |\vec{F}_y| = m_1 g \cos \theta.
   \]

   \[
   \sum F_z = m_1 a_z \Rightarrow |\vec{F}_z| - |\vec{F}_y| \sin \theta = 0.
   \]

   At mass center,
   \[
   |\vec{F}_x| = \mu_s |\vec{F}_z|, \quad \text{so}
   \]

   \[
   \mu_s |\vec{F}_z| - |\vec{F}_y| = m_1 g \sin \theta = 0 \ldots \quad \text{plug in earlier results...}
   \]

   \[
   \mu_s m_1 g \cos \theta - m_2 g \sin \theta - m_1 g \sin \theta = 0
   \]

   \[
   \mu_s m_1 \cos \theta = (m_1 + m_2) g \sin \theta
   \]

   \[
   \tan \theta = \frac{\mu_s m_1}{(m_1 + m_2) g} = \frac{0.18(1000 \text{ kg})}{(1500 \text{ kg})}
   \]

   \[
   = 0.12, \quad \text{i.e.,} \quad 12^\circ \text{ grade.}
   \]

   So, 10^\circ is OK.

4. Evaluation
   - Correct sign?
   - Appropriate units?
   - Reasonable magnitude?
Constrained Motion Problem Worksheet

1. Pictorial Representation
   - coordinate axes
   - sketch of situation, labeled with symbols
   - $x$ is the direction that the string pulls on $m$.

List known quantities:
- $m, M, r, g$ one do be taken as "given," even without numerical values.
- List unknown quantities:
  - $\vec{v}$ for mass on table.

2. Physical Representation
   - describe the constraint: uniform circular motion for $m, M$ and $r$
   - force diagram for each object, listing agent for each force
   - net force points in direction of acceleration

3. Mathematical Representation
   - For hanging mass:
     \[ \sum F_y = Ma_y \]
     \[ \rightarrow |F_T| - 1g |F_T| = 0 \]
     \[ \rightarrow |F_T| = Mg. \]
   - For mass on table:
     \[ \sum F_x = ma_x \]
     \[ \rightarrow |F_T| = m \frac{v^2}{R} \]
     \[ \rightarrow \frac{R}{m} |F_T| = 1v^2 \]
     \[ \rightarrow 1v = \sqrt{\frac{R}{m} |F_T|} \]

By Newton's 3rd law, tension forces are equal in magnitude.

So...
\[ |v| = \sqrt{\frac{M}{m} Rg}. \]

This makes sense: increase $M, R$ or $g$ and speed must be greater.
Increase $m$, and it can be less.

4. Evaluation
   - correct sign?
   - appropriate units?
   - reasonable magnitude?
     - say $M = m, \ R = 5 \ m$
     \[ |v| = \sqrt{\frac{5}{5}} \ m/s = 2.2 \ m/s. \]
\#4

\[ X_{cm} = \frac{X_e m_e + X_m m_m}{m_e + m_m} = \frac{0 + R m_m}{m_e + m_m} = \frac{(3.81 \times 10^6 \text{ m})(17.36 \times 10^3 \text{ kg})}{5.78 \times 10^4 \text{ kg} + 7.16 \times 10^3 \text{ kg}} = 4.6 \times 10^6 \text{ m} \]

\[ \frac{X_{cm}}{r_e} = \frac{4.6 \times 10^6 \text{ m}}{6.4 \times 10^6 \text{ m}} = 0.73 \]

\#5

\[ X_{cm} = \frac{X_e m_e + X_o m_o}{m_e + m_o} = \frac{0 + (16u) X_o}{(12u) + (16u)} = \frac{16}{28} X = \frac{16}{28} \cdot (1.13 \times 10^6 \text{ m}) \]

(Note that \( X_{cm} \) is a bit closer to the \( O \).) = 6.46 \times 10^6 \text{ m}

\#7

\[ \text{Initial:} \]

\[ \text{Final:} \]

\[ m_c = 58 \text{ kg} \]

\[ m_b = 30 \text{ kg} \] (mass of ball)

\[ m_R = ? \]

For an isolated system, initially at rest, the CM remains at rest.

If the canoe moves 40 cm from the fixed CM, it must lie 20 cm on one side initially and 20 cm on the other side afterwards.

Therefore, taking the CM to be \( x = 0 \), \( x_e = -1.3 \text{ m} \), \( x_b = 1.2 \text{ m} \), \( x_c = 1.7 \text{ m} \).

But \[ X_{cm} = \frac{m_R x_R + m_c x_c + m_b x_b}{m_{\text{total}}} \] (initial)

\[ 0 = m_R (-1.3 \text{ m}) + m_c (1.7 \text{ m}) + m_b (1.2 \text{ m}) \]

\[ m_R (1.3 \text{ m}) = m_c (1.7 \text{ m}) + m_b (1.2 \text{ m}) \]

\[ m_R = \frac{(58 \text{ kg})(1.7 \text{ m}) + (30 \text{ kg})(1.2 \text{ m})}{1.3 \text{ m}} = 80 \text{ kg} \]
Cyrano de Bergerac (see the play) is lying. His method may indeed lift him and the plate off the ground momentarily (if it's a strong magnet). But once in the air, the only outside force on the system (plate + Cyrano + magnet) is gravity, and therefore the system's center of mass must accelerate downward at 9.8 m/s². At this point he'd better off throwing magnet and plate downward — then at least he'd recoil up a little farther. But he'll never make it to the moon in any case.

10. A system's momentum is always conserved unless there are external forces acting on it. If I crash my car into a building, the total momentum of car + building is not conserved because of the external force anchoring the building to the ground. However, by enlarging the "system" to include the entire earth, we can still say that momentum is conserved.
8. \[ \begin{align*}
V_x &= \frac{m}{
u} \cos \theta = (1000 \text{ kg})(55 \text{ mph}) \left( \frac{1 \text{ m/s}}{2.24 \text{ mph}} \right) \cos 25^\circ \\
&= 22,250 \text{ kg m/s} = 22,000 \text{ kg m/s} \\
V_y &= -m \nu \sin \theta = -(1000 \text{ kg})(55 \text{ mph}) \left( \frac{1 \text{ m/s}}{2.24 \text{ mph}} \right) \sin 25^\circ \\
&= -10,400 \text{ kg m/s}.
\end{align*} \]

9. If we're going the same direction, then for no momentum to equal the car:
\[ m_\text{me} \cdot V_\text{me} = m_\text{car} \cdot V_\text{car} \]
\[ \rightarrow (80 \text{ kg}) \frac{V_\text{me}}{} = (1600 \text{ kg})(1.2 \text{ km/hr}) \]
\[ \rightarrow V_\text{me} = (1.2 \text{ km/hr}) \cdot \frac{1600}{80} \]
\[ = 1.2 \text{ km/hr} \cdot 20 = 24 \text{ km/hr} \]
(That's 6.7 mph.)
Conservation Laws Worksheet  
(Child and Rock)

(Adapted from Van Heuvelen, 
Overview Case-Study Physics)

1. Pictorial Representation

- Symbols, including masses and velocities
- Coordinate axes

<table>
<thead>
<tr>
<th>INITIAL</th>
<th>FINAL</th>
</tr>
</thead>
<tbody>
<tr>
<td><img src="image1" alt="Initial Diagram" /></td>
<td><img src="image2" alt="Final Diagram" /></td>
</tr>
</tbody>
</table>

List known quantities:
- \( M = 40 \text{ kg} \)
- \( m = 2 \text{ kg} \)
- \( |\vec{v}_r| = 8 \text{ m/s} \)

List unknown quantities:
- \( \vec{v}_c \)

2. Physical Representation

Specify what the "system" is: **Child plus rock**

Which conservation law(s) apply, and why? **Outside forces (gravity, normal force) don’t have time to act on rock, since we’ve given its speed just after being thrown. These forces cancel for the child. Thus momentum is conserved.**

3. Mathematical Representation

Write the appropriate conservation law(s) in mathematical form and solve.

\[
P_{x, f} = P_{x, i}
\]

\[
\Rightarrow -M|\vec{v}_e| + m|\vec{v}_e'| = 0
\]

\[
|\vec{v}_e'| = \frac{m}{M} |\vec{v}_e| = \frac{2 \text{ kg}}{40 \text{ kg}} \cdot 8 \text{ m/s}
\]

\[
= 0.4 \text{ m/s}
\]

The child recoils to the west at \(0.4 \text{ m/s} \).

4. Evaluation

- Correct sign?
- Appropriate units?
- Reasonable magnitude?
Conservation Laws Worksheet

1. Pictorial Representation
   - Symbols, including masses and velocities
   - Coordinate axes
   
   INITIAL
   \[ m_1 \rightarrow V_1 \rightarrow x \]
   \[ m_2 \rightarrow \]
   
   FINAL
   \[ m_1 + m_2 \rightarrow V_f \rightarrow x \]
   
   List known quantities:
   - \( m_1 = 32 \text{ tons} \)
   - \( m_2 = 24 \text{ tons} \)
   - \( V_1 = 5 \text{ ft/s} \)
   - \( V_2 = 3 \text{ ft/s} \)
   
   List unknown quantities:
   - \( V_f = ? \)

2. Physical Representation
   Specify what the "system" is: The 2 railroad cars
   Which conservation law(s) apply, and why? Assuming the track is horizontal and friction with the track is negligible, the net external force is zero, so the system's total momentum is conserved.

3. Mathematical Representation
   Apply the appropriate conservation law(s) in mathematical form and solve.
   \[ \sum P_{x f} = \sum P_{x i} \]
   \[ \Rightarrow (m_1 + m_2)V_f = m_1 V_1 + m_2 V_2 \]
   \[ \Rightarrow V_f = \frac{m_1 V_1 + m_2 V_2}{m_1 + m_2} \]
   \[ = \frac{(32 \text{ tons})(5 \text{ ft/s}) + (24 \text{ tons})(3 \text{ ft/s})}{32 + 24} \text{ ft/s} \]
   \[ = \frac{160 + 72}{56} \text{ ft/s} \]
   \[ = 4.1 \text{ ft/s} \]
   
   (Note that there's no need to convert to SI units.)

4. Evaluation
   - Correct sign?
   - Appropriate units?
   - Reasonable magnitude?
First consider the beanbag. It hits the pin with a thrust, comes to rest, and thus transfers essentially all of its momentum to the pin:

\[ \vec{p}_i \rightarrow \vec{p}_f = 0 \]

Now consider the rubber ball. It bounces back with about the same speed:

\[ \vec{p}_i \rightarrow \vec{p}_f \rightarrow \vec{p}_f - \vec{p}_i \]

Since the ball's momentum changes by twice as much in this case, the pin's momentum must also change by twice as much as it acquires a momentum in the direction of the ball's original motion.

So choose the rubber ball!
Conservation Laws Worksheet

1. **Pictorial Representation**
   - Symbols, including masses and velocities
   - Coordinate axes

   **INITIAL**
   - $m_a$, $m_b$
   - $v_a$, $v_b$

   **FINAL**
   - $v_f$

   List known quantities:
   - $m_a = 2700$ lb
   - $m_b = 3600$ lb
   - $v_a = 40$ mi/hr
   - $v_b = 60$ mi/hr

   List unknown quantities:
   - $v_f$
   - (magnitude and direction)

2. **Physical Representation**
   Specify what the "system" is:
   - The two cars.

   Which conservation law(s) apply, and why?
   - Momentum is approximately conserved in the collision itself, though probably not for long after.

3. **Mathematical Representation**
   Apply the appropriate conservation law(s) in mathematical form and solve.

   \[
   P_{x_i} = P_{x_f} \\
   \rightarrow m_a(-v_a) + m_b = (m_a + m_b)v_f \times \frac{m_a}{m_a + m_b}
   \]

   \[
   \rightarrow v_{fx} = \frac{-m_a}{m_a + m_b} v_a = \frac{-2700}{2700 + 3600} (40 \text{ mph}) = -17.1 \text{ mph}
   \]

   \[
   P_{y_i} = P_{y_f} \\
   \rightarrow 0 + m_b(-v_b) = (m_a + m_b)v_{fy} \frac{m_b}{m_a + m_b}
   \]

   \[
   \rightarrow v_{fy} = \frac{-m_b}{m_a + m_b} v_b = \frac{-3600}{2700 + 3600} (60 \text{ mph}) = -34.3 \text{ mph}
   \]

   \[
   \text{speed} = \sqrt{v_{fx}^2 + v_{fy}^2} = 38 \text{ mph}
   \]

4. **Evaluation**
   - correct sign?
   - appropriate units?
   - reasonable magnitude?