Solutions to Problem Set 15

(Physics 2210, Prof. Schroeder)

1. (a) When the gas expands, each molecule has more possible places to be, so the number of possible states of the system (i.e., the multiplicity) increases.
(b) When the temperature of a gas increases, the average energy per molecule increases, so the gas has more total energy. With more energy in the system, there are more ways of arranging the energy among the molecules, so the multiplicity of possible states is greater. (Ridiculously simple example: If two molecules are sharing one unit of energy, then either molecule can have it, so there are two possible arrangements; if they're sharing two units of energy, then there are three possible arrangements, since one or the other molecule could have both units, or they could each have one unit.)
(c) In an ice cube, the water molecules are locked into a crystal lattice so they're not free to move very far or to change their orientations. When the ice cube melts, however, the molecules are free to rotate in any direction as well as move large distances. So the molecules in liquid water have a much greater multiplicity of arrangements than they had in the solid ice.
(d) The fall itself doesn't increase Humpty Dumpty's multiplicity, but when he lands and breaks into pieces, there are many possible ways he could break and many possible ways of scattering the pieces on the ground, so his multiplicity increases greatly. And unfortunately, all the king's horses and all the king's men can't violate the second law of thermodynamics!

2. $$\Delta S_{\text{boiling}} = \frac{Q}{T} = \frac{mL}{T} = \frac{(250\text{ g})(2260\text{ J/g})}{373\text{ K}} = 1515\ \frac{\text{J}}{\text{K}} \approx \underline{1500\ \frac{\text{J}}{\text{K}}}.$$

$$\Delta S_{\text{freezing}} = \frac{Q}{T} = \frac{-mL}{T} = \frac{-(250\text{ g})(333\text{ J/g})}{273\text{ K}} = -305\ \frac{\text{J}}{\text{K}} \approx \underline{-300\ \frac{\text{J}}{\text{K}}}.$$

(negative because the heat <u>leaves</u> the system)

3. (a) $\Delta S_{\text{outdoors}} = \frac{Q}{T} = \frac{15\times10^6\,J}{273\,K} = \underline{55,000\,\frac{J}{K}}$ (positive).

(b) $\Delta S_{\text{indoors}} = \frac{Q}{T} = \frac{-15\times10^6\,J}{298\,K} = \underline{-50,300\,\frac{J}{K}}$ (negative).

(c) $\Delta S_{\text{net}} = +55,000\,\frac{J}{K} - 50,300\,\frac{J}{K} = \underline{+4600\,\frac{J}{K}}$.

Indeed, the total entropy of the universe increases.

5. (a) $\Delta S_{\text{from heat input}} = \frac{Q}{T} = \frac{10^6\,J}{673\,K} = 1486\,\frac{J}{K}$, or more if the engine's temp. is less than 400°C.

$\approx \underline{1500\ J/K}$

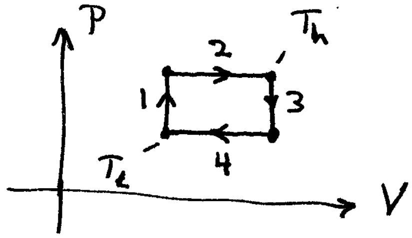
(b) $Q = T\cdot\Delta S_{\text{output}} = (298\,K)(1486\,\frac{J}{K}) = 443,000\ J \approx \underline{4.4\times10^5\ J}$.

(c) $1,000,000\ J - 443,000\ J = 557,000\ J \approx \underline{5.6\times10^5\ J}$.

(d) $\text{efficiency} = \frac{W}{Q_{\text{input}}} = \frac{5.6\times10^5\,J}{10^6\,J} = \underline{.56}$.

or $1 - \frac{T_c}{T_h} = 1 - \frac{298\,K}{673\,K} = \underline{.56}$. ✓

4. (a)



The initial volume is

$$V_i = \frac{nRT}{P} = \frac{(1\text{ mole})(8.31\frac{J}{\text{mol}\cdot K})(300K)}{10^5\ N/m^2}$$

$$= \underline{.0249\ m^3}$$

(b) $\boxed{W}$

Step 1: $W = -P\cdot\Delta V = 0.$

Step 2: $W = -P\cdot\Delta V = -(2\times10^5\frac{N}{m^2})(.0249\ m^3) = -5000\ J.$

Step 3: $W = -P\cdot\Delta V = 0.$

Step 4: $W = -(10^5\frac{N}{m^2})(-.0249\ m^3) = +2500\ J.$

$\boxed{\Delta U}$

Step 1: $\Delta E = \frac{5}{2}nR\,\Delta T = \frac{5}{2}V\cdot\Delta P = \frac{5}{2}(.025\ m^3)(10^5\frac{N}{m^2}) = 6250\ J.$

Step 2: $\Delta E = \frac{5}{2}nR\Delta T = \frac{5}{2}P\cdot\Delta V = \frac{5}{2}(2\cdot10^5\frac{N}{m^2})(.025\ m^3) = 12{,}500\ J.$

Step 3: $\Delta E = \frac{5}{2}V\cdot\Delta P = \frac{5}{2}(.05\ m^3)(-10^5\frac{N}{m^2}) = -12{,}500\ J.$

Step 4: $\Delta E = \frac{5}{2}P\cdot\Delta V = \frac{5}{2}(10^5\frac{N}{m^2})(-.025\ m^3) = -6250\ J.$

$\boxed{Q}$

Step 1: $Q = \Delta E + W = 6250\ J$

Step 2: $Q = \Delta E + W = 17{,}500\ J.$

Step 3: $Q = \Delta E + W = -12500\ J.$

Step 4: $Q = \Delta E + W = -8750\ J.$

(c) Total work $= -5000\ J + 2500\ J = \underline{-2500\ J}.$

Total $\Delta E = 6250\ J + 12500\ J - 12500\ J - 6250\ J = \underline{0}.$

Total $Q = 6250\ J + 17500\ J - 12500\ J - 8750\ J = \underline{2500\ J}$

(d) $\text{efficiency} = \frac{|W_{net,out}|}{Q_{input}} = \frac{+2500\ J}{6250\ J + 17500\ J} = .105 = \underline{10.5\%}$

(e) T_H is at upper-right corner, where P, V are both twice what they are at lower-left. Since $PV = Nk_BT$, this means T_H is $\underline{4\times}$ T_L, so the ideal Carnot efficiency would be $1 - \frac{T_L}{T_H} = 1 - \frac{1}{4} = \underline{75\%}.$

6. (a) max efficiency $= 1 - \frac{T_c}{T_H} = 1 - \frac{293\text{ K}}{773\text{ K}} = \underline{.621}$

(b) e = efficiency $= \frac{W_{net}}{Q_{input}} = \frac{W_{net}}{W_{net} + Q_{out}}$

$\rightarrow W_{net} + Q_{out} = \frac{W_{net}}{e} \rightarrow Q_{out} = W_{net}\left(\frac{1}{e} - 1\right)$

$= W_{net} \cdot \left(\frac{1}{.62} - 1\right) = (.61) W_{net}$.

During 1 second, $W_{net} = 10^9$ J, and $Q_{out} = .61 \times 10^9$ J,

i.e., heat is dumped to the river at a rate of $\underline{.61\text{ GW}}$.

This heat is distributed among 10^5 liters or 10^5 kg of water.

$Q = mc \cdot \Delta T \rightarrow \Delta T = \frac{Q}{mc} = \frac{.61 \times 10^9\text{ J}}{(10^5\text{ kg})(4186\text{ J/kg})}$

$= \underline{1.5\text{ K}}$ or $\underline{1.5\ ^\circ\text{C}}$.

(c) Now max efficiency is $1 - \frac{293\text{ K}}{873\text{ K}} = 66.4\%$

So for a given Q_{input}, W_{net} is greater by the ratio $\frac{664}{621} = 1.069$ or about 7%.

Thus we get an extra $.07 \times 10^9$ J/s.

$\Delta\$ = \left(.07 \frac{\text{GJ}}{\text{s}}\right)\left(\frac{3.16 \times 10^7\text{ s}}{\text{year}}\right)\left(\frac{1\text{ kw-hr}}{3.6 \times 10^6\text{ J}}\right)\left(\frac{.05\ \$}{1\text{ kw-hr}}\right)$

$= 3 \times 10^7$ \$/yr $=$ $\underline{30\text{ millions dollars}}$.

8. As computed in class, max $COP = \frac{T_L}{T_H - T_L} = \frac{275\,K}{25\,K}$

$= \underline{11}$.

But $COP = \frac{\text{benefit}}{\text{cost}} = \frac{Q_{in}}{W}$,

so $Q_{in} = 11 \cdot W$ or $W = \frac{Q_c}{11} = \frac{200\ J}{11}$

$= 18.2\ J$ in one second.

Thus on average, this <u>ideal</u> refrigerator would need to consume only 18 watts of electrical power.

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7. My kitchen might cool off at first, as the air from the fridge comes out and mixes. But then the fridge will run <u>more</u>, using more power from the wall and converting this work into heat that's dumped into the kitchen. Remember: $Q_h = Q_c + W$ by energy conservation, so the fridge always dumps more heat into the kitchen than it removes from "inside".