

# Solutions to Problem Set 14

Physics 221b  
Schroeder

1. Suppose I have 200 g water, initially at 20°C.

$$\begin{aligned} \text{Then (Energy supplied)} &= mc \cdot \Delta T = (200 \text{ g}) \left( 1 \frac{\text{cal}}{\text{g} \cdot ^\circ\text{C}} \right) (80^\circ\text{C}) \\ &= \underline{16,000 \text{ cal}} \times \left( \frac{4.2 \text{ J}}{\text{cal}} \right) = \underline{67,000 \text{ J}}. \end{aligned}$$

$$\begin{aligned} \text{At } 600 \frac{\text{J}}{\text{s}}, \text{ this would take } &\frac{67,000 \text{ J}}{600 \frac{\text{J}}{\text{s}}} = 110 \text{ s} \\ &= \underline{2 \text{ minutes}}. \end{aligned}$$

But this energy transfer is not a spontaneous one from a hotter object to a cooler one, so technically the microwaves do not "heat" the water, they do work on it. (If anything, heat flows out of the water to the surroundings.)

$$2(a) \ c = \frac{\text{Energy supplied}}{M \cdot \Delta T} = \frac{314 \text{ J}}{(30 \text{ g})(10^\circ\text{C})} = \underline{1.05 \frac{\text{J}}{\text{g} \cdot ^\circ\text{C}}}.$$

$$(b) \ n = \frac{\text{total mass}}{\text{molar mass}} = \frac{30 \text{ g}}{50 \text{ g}} = \underline{0.6 \text{ moles}}.$$

$$(c) \ C = \frac{\text{Energy supplied}}{n \cdot \Delta T} = \frac{314 \text{ J}}{(0.6)(10^\circ\text{C})} = \underline{52 \text{ J/mol}}.$$

3. (a) Potential energy gained =  $mgh$ ,  
 but 25% efficient so 4x as much  
 energy needed:

$$\begin{aligned} m &= 75 \text{ kg} \\ g &= 10 \text{ N/kg} \\ h &= 9572' - 4700' \\ &= 4872' = \underline{1484 \text{ m}} \end{aligned}$$

$$\begin{aligned} 4mgh &= 4(75 \text{ kg})(10 \frac{\text{N}}{\text{kg}})(1484 \text{ m}) \\ &= (4.4 \times 10^6 \text{ J}) \left( \frac{1 \text{ cal}}{4.186 \text{ J}} \right) \left( \frac{1 \text{ kcal}}{1000 \text{ cal}} \right) \left( \frac{1 \text{ bcf}}{100 \text{ kcal}} \right) \\ &= 10.4 \text{ bcf of corn flakes} \\ &\quad \text{(rounded to } \underline{10.}) \end{aligned}$$

(b)  $\frac{3}{4}$  of the 10.4 bcf's becomes thermal energy:

$$7.8 \text{ bcf's, or } 780 \text{ kcal.}$$

Assuming the professor is mostly made of water,

$$\Delta T = \frac{(780 \text{ kcal})}{(75 \text{ kg})(1 \text{ kcal/kg}^{\circ}\text{C})} = 10.4 \text{ }^{\circ}\text{C} \approx \underline{10^{\circ}\text{C}} \quad \ddot{=}$$

(Fatal!)

(c) The same 780 kcal could evaporate

$$\frac{780 \text{ kcal}}{580 \text{ kcal/kg}} = 1.3 \text{ kg or } \underline{1.3 \text{ liters}} \text{ of water.}$$

4. (a) I'm not sure whether all the ice melts. But suppose it doesn't. Then the final temp. is  $0^\circ\text{C}$ , so the heat lost by the water is

$$Q_w = m_w c_w \Delta T = (180 \text{ g}) \left(1 \frac{\text{cal}}{\text{g}\cdot^\circ\text{C}}\right) (30^\circ\text{C}) = 5400 \text{ cal.}$$

The heat lost by the beaker is

$$Q_b = m_b c_b \Delta T = (100 \text{ g}) \left(0.215 \frac{\text{cal}}{\text{g}\cdot^\circ\text{C}}\right) (30^\circ\text{C}) = \underline{645 \text{ cal}}$$

$$\text{total heat avail.} = 6045 \text{ cal.}$$

Melting ice requires  $80 \text{ cal./gram}$ , so with  $6045 \text{ cal}$  we can melt  $\frac{6045}{80} \text{ g} = 76 \text{ g}$  of ice.

Thus  $24 \text{ g}$  remains.

(b) Now there's definitely enough heat to melt all the ice, so the final temp.,  $T_f$ , is unknown.

The heat lost by the water is  $Q_w = m_w c_w (30^\circ - T_f)$ .

The heat lost by the beaker is  $Q_b = m_b c_b (30^\circ - T_f)$ .

The heat gained by the ice is  $Q_i = m_i L + m_i c_w (T_f - 0^\circ\text{C})$

Equate heat lost to heat gained:

$$(m_w c_w + m_b c_b)(30^\circ - T_f) = m_i L + m_i c_w (T_f - 0^\circ\text{C})$$

$$\Rightarrow m_i c_w T_f + (m_w c_w + m_b c_b) T_f = (m_w c_w + m_b c_b)(30^\circ\text{C}) - m_i L$$

$$\Rightarrow T_f = \frac{(m_w c_w + m_b c_b)(30^\circ\text{C}) - m_i L}{m_w c_w + m_b c_b + m_i c_w} = \frac{(180 \text{ g} \cdot 1 \frac{\text{cal}}{\text{g}\cdot^\circ\text{C}} + 100 \text{ g} \cdot 0.215 \frac{\text{cal}}{\text{g}\cdot^\circ\text{C}})(30^\circ\text{C}) - (50 \text{ g})(80 \frac{\text{cal}}{\text{g}})}{180 \text{ g} \cdot 1 \frac{\text{cal}}{\text{g}\cdot^\circ\text{C}} + 100 \text{ g} \cdot 0.215 \frac{\text{cal}}{\text{g}\cdot^\circ\text{C}} + 50 \text{ g} \cdot 1 \frac{\text{cal}}{\text{g}\cdot^\circ\text{C}}} = \underline{8.1^\circ\text{C}}$$

$$5. (a) W = -P \cdot \Delta V = -(1 \text{ atm})(2 \text{ L} - 9 \text{ L}) = + 7 \text{ L} \cdot \text{atm}$$

convert units:  $\times \left( \frac{10^5 \text{ Pa}}{1 \text{ atm}} \right) \left( \frac{1 \text{ m}^3}{1000 \text{ L}} \right) = \underline{560 \text{ J}}$ .

$$(b) \Delta E = Q + W = (-400 \text{ J}) + (560 \text{ J}) = \underline{+160 \text{ J}}$$

↑  
negative since heat leaves.

$$6. W = -P \cdot \Delta V = -P(V_f - V_i), \text{ but } V_i \text{ is negligible compared to } V_f$$

$$\approx -PV_f = -nRT$$

$$= -(1 \text{ mol}) \left( \frac{8.31 \text{ J}}{\text{mol} \cdot \text{K}} \right) (373 \text{ K}) = -3100 \text{ J}$$

So the work done by the steam is +3100 J.

$$\Delta E = Q + W$$

$$= ML + W$$

$$= (18 \text{ g}) \left( 2260 \frac{\text{J}}{\text{g}} \right) + (-3100 \text{ J}) \quad \text{since } 1 \text{ mol} = 18 \text{ g}$$

$$= \underline{37,600 \text{ J}}$$

Of the 40,700 J put in as heat, only 37,600 J stays in the system. The other 3100 J leave in the form of work done on the surroundings.

$$\begin{aligned}
7. \quad W &= -(\text{area under } P\text{-}V \text{ graph}) \\
&= -\int_{V_i}^{V_f} P \, dV = -\int_{V_i}^{V_f} \frac{nRT}{V} \, dV = -nRT \int_{V_i}^{V_f} \frac{dV}{V} \\
&= -nRT \ln V \Big|_{V_i}^{V_f} = -nRT (\ln V_f - \ln V_i) \\
&= -nRT \ln \left( \frac{V_f}{V_i} \right) = -P_i V_i \ln \frac{V_f}{V_i} = -(10^5 \frac{N}{m^2})(0.01 \text{ m}^3) \\
&\quad \times \ln \left( \frac{1}{10} \right) \\
&= \underline{+ 2330 \text{ J}}.
\end{aligned}$$

$\Delta E = 0$  since  $T$  is constant and for an ideal gas,  $E$  depends only on  $T$ .

So  $Q = \Delta E - W = -W = \underline{-2330 \text{ J}}$ , that is, heat leaves the gas.

8. For each molecule,

$$\begin{aligned}
\overline{KE}_{\text{trans}} &= \frac{3}{2} k_B T = \frac{3}{2} (1.38 \times 10^{-23} \frac{\text{J}}{\text{K}})(290 \text{ K}) \\
&= \underline{6.0 \times 10^{-21} \text{ J}} \times \left( \frac{1 \text{ eV}}{1.6 \times 10^{-19} \text{ J}} \right) = \underline{.038 \text{ eV}}.
\end{aligned}$$

$$\begin{aligned}
\text{Total } KE_{\text{trans}} &= N \cdot \overline{KE} \approx (2 \times 10^{27})(6 \times 10^{-21} \text{ J}) \\
&= 12 \times 10^6 \text{ J} = \underline{12 \text{ MJ}}.
\end{aligned}$$

9.  $\bar{K}_{\text{trans}} = \frac{3}{2} k_B T$ , independent of  $m$ , so the molecules have the same average translational kinetic energy.

$v_{\text{rms}} = \sqrt{\frac{3k_B T}{m}}$ , so the lighter molecules ( $\text{H}_2$ ) are moving faster, by a factor of

$$\sqrt{\frac{m_{\text{O}_2}}{m_{\text{H}_2}}} = \sqrt{\frac{32}{2}} = \sqrt{16} = \underline{4}.$$

10. The equipartition theorem says  $\bar{E}_{\text{molecule}} = \frac{f}{2} k_B T$ , so for the whole gas,  $E = \frac{f}{2} N k_B T = \frac{f}{2} n R T$ .


The molar specific heat is  $\frac{\Delta E}{n \Delta T} = \frac{1}{n} \frac{dE}{dT} = \frac{1}{n} \cdot \frac{f}{2} n R = \frac{f}{2} R$ .

$$\text{So } C = \frac{f}{2} R \rightarrow f = \frac{2C}{R} = \frac{2(27 \text{ J/mol}\cdot\text{K})}{8.31 \text{ J/mol}\cdot\text{K}} = \underline{6.5}.$$

What are the degrees of freedom?

3 translational ( $v_x, v_y, v_z$ )

3 rotational ( $\omega_x, \omega_y, \omega_z$ )

Many vibrational, but most are "frozen out" at room temperature. The easiest vibrational mode to excite is "flexing", . It should count as 2 degrees of freedom (kinetic plus potential energy), but apparently only counts as  $\frac{1}{2}$  since it's mostly frozen out.

11. (a) At constant  $V$ ,  $W = -P \Delta V = 0$  so  $\Delta E = Q$ .

$$\begin{aligned} \text{So } Q &= E_f - E_i = N \cdot \frac{f}{2} k_B T_f - N \cdot \frac{f}{2} k_B T_i = N \cdot \frac{f}{2} k_B \Delta T \\ &= nR \cdot \frac{f}{2} \Delta T = (2 \text{ mol}) (8.31 \frac{\text{J}}{\text{mol} \cdot \text{K}}) \cdot \frac{5}{2} \cdot (20^\circ \text{C}) \\ &= \underline{831 \text{ J}}. \end{aligned}$$

(b) At constant  $P$ , the gas expands and performs

$$\begin{aligned} \text{work: } W &= -P \cdot \Delta V = -(P V_f - P V_i) = -(nRT_f - nRT_i) \\ &= -nR \cdot \Delta T = -(2 \text{ mol}) (8.31 \frac{\text{J}}{\text{mol} \cdot \text{K}}) (20^\circ \text{C}) \\ &= -332 \text{ J}. \end{aligned}$$

$$\text{So } Q = \Delta E - W = (831 \text{ J}) - (-332 \text{ J}) = \underline{1160 \text{ J}}.$$

12. Being lazy, I should compress the gas slowly, then heat will leak out so it won't get so hot, so it will be at a lower average pressure:

$$P = \frac{N k_B T}{V}, \quad W = -P \cdot \Delta V$$

If I compress it quickly, the work I do will add energy to the gas and it will get hot, hence build up more pressure, requiring me to do more work.